4a) In Δ's ABC & ACH we have

H is the orth. projection of C on [AB] (given)

So, $C\hat{H}A = 90°$.

but, $A\hat{C}B = 90°$ (proved).

then, $A\hat{H}C = A\hat{C}B = 90°$ (by comparison)

And, $C\hat{A}H$ is a common angle

Thus, Δ's are similar by A.A postulate (Two angles in 1st Δ are resp. equal to two angles in the 2nd Δ).

Similarity Ratios:

Δ ACB	$\frac{AC}{AH}$	$\frac{CB}{HC}$	$\frac{AB}{AC}$ = cst.
Δ AHC	①	②	③

b) Using ratios ① & ③:

$\frac{AC}{AH} = \frac{AB}{AC}$ ---ⓐ

but, $\cos H\hat{A}C = \frac{adj}{hyp}$

So, $\cos 50° = \frac{AH}{AC}$

then, $AC = \frac{AH}{\cos 50°}$ ---ⓑ

replace ⓑ in ⓐ:

$\frac{AH}{AH\cos 50°} = \frac{AB}{\frac{AH}{\cos 50°}}$

So, $\frac{1}{\cos 50°} = \frac{AB\cos 50°}{AH}$

Thus, $AH = AB\cos^2 50°$.

$= 8\cos^2 50°$ cm

H belongs to [AO] (given)

So, $HO = AO - AH$.

$= 4 - 8\cos^2 50°$ cm.