4^{th} exercise :

1) (on graph)

2) a. In Δs OFH & OKA we have:

(O, F, K) & (O, H, A) collinear (given).

Apply Converse of Thales' Theorem : a st line dividing 2 sides of Δ prop. is (//) to 3^{rd} side.

Ratios: $\frac{OE}{EK} \overset{?}{=} \frac{OH}{HA}$

$\frac{R}{R} \overset{?}{=} \frac{\frac{R}{2}}{\frac{R}{2}}$ (by symetry: $FO = FK = R$adius; by midpoint: $HO = HA = \frac{\text{Radius}}{2}$).

$1 = 1$.

Then, (FH) is parallel to (KA)

But, (FH) perpendicular bisector of [AO] (given).

Then, (KA) perpendicular to [AO] at A (2 parallel line on perpen. to same st line).

Thus, (KA) is tangent to (C) at A

((KA) $\perp$ radius converse of tangent theorem)

b. In KAO we have:

~~Apply Converse of Pythagoras Theorem:~~

~~$KO^2 \overset{?}{=} KA^2 + AO^2$~~

~~$6^2 = KA^2 + 3^2$~~

K symmetric of O w.r.t. F (given)

F midpoint of [KO]

But, FO = AO = Radius

& FO = KF (midpoint)

Then, median = $\frac{\text{longest}}{2}$ $AE = \frac{KO}{2} = \frac{6}{2} = 3$

Then, by converse of median relative to hyp theorem:

KAO is a right Δ at A.

So, (KA) $\perp$ (AO) (right angle of Δ).

Thus, (KA) is tangent to (C) at A (1).

AE = FO (F on perpen bisector of [AO]) Equilateral (for EOU).

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