$V = \frac{-\sqrt{6}-\sqrt{2}}{4}$ is a solution of given eqn if it satisfies it.

$$4\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)^2 + 2\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)(\sqrt{2}) - 1 \overset{?}{=} 0$$

note: $(-a-b)^2 = (a+b)^2$
$= (a^2+b^2+2ab$.

$$4\left(\frac{8+4\sqrt{3}}{16}\right) - 2\left(\frac{-2\sqrt{3}-2}{4}\right) - 1 \overset{?}{=} 0$$

$$16\left(\frac{2+\sqrt{3}}{16}\right) + 4\frac{(\sqrt{3}+1)}{4} - 1 \overset{?}{=} 0$$

$$2+\sqrt{3} + \sqrt{3}+1-1 \overset{?}{=} 0$$

$$0 = 0$$

Thus, verified.

b) Since, $\cos 75^\circ = x$ verifies $4x^2+2x-1=0$ (given)
& U & V verify $4x^2+2x-1=0$ proved

then $\cos 75^\circ = U$ or $\cos 75^\circ = V$
but 75° is an acute angle
so, $0 < \cos 75 < 1$
but $0 < U < 1$
& $V < 0$
so U is accepted and V is rejected
Thus $\cos 75^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos^2 75^\circ + \sin^2 75^\circ = 1$ (pythagorean identity)

so, $\sin^2 75^\circ = 1 - \cos^2 75$

$$= 1 - \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2$$

$$= 1 - \frac{(8-4\sqrt{3})}{16}$$

$$= 1 - \frac{4(2-\sqrt{3})}{16}$$

$\sin^2 75^\circ = \frac{4-2+\sqrt{3}}{4}$

$= \frac{2+\sqrt{3}}{4}$

hence, $\sin 75^\circ = \mp\frac{(\sqrt{2+\sqrt{3}})}{2}$.

but $0 < \sin 75^\circ < 1$ since 75° is acute.

Thus, $\sin 75^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}$ accep.