to find a relation between MO and MN, we find their ratio:

$$\frac{MO}{MN} = \frac{2\sqrt{5}-2}{-1+\sqrt{5}} = \frac{2(\sqrt{5}-1)}{\sqrt{5}-1} = 2$$

so, $MO = 2MN$ (proved)

2) a) $MO^2 = (2\sqrt{5}-2)^2$
$= 20 - 8\sqrt{5} + 4$
$= 24 - 8\sqrt{5}$ cm

$MN^2 = (\sqrt{5}-1)^2$
$= 5 - 2\sqrt{5} + 1$
$= 6 - 2\sqrt{5}$ cm

$NO^2 = (\sqrt{15}-\sqrt{3})^2$
$= 15 - 2\sqrt{3^2 \times 5} + 3$
$= 18 - 6\sqrt{5}$ cm

b) $MO^2 \overset{?}{=} MN^2 + NO^2$

since $24 - 8\sqrt{5} \overset{?}{=} 6 - 2\sqrt{5} + 18 - 6\sqrt{5}$
$24 - 8\sqrt{5}\,\text{cm} = 24 - 8\sqrt{5}$ cm

Thus, MON is right at N (converse of Pythagorean theorem)

- but MO = 2MN (proved)

So hyp = 2MN

Thus, MON is semi-equilateral at N since it is right having its hypotenuse the double of one of its sides (proved)

3) a) area MON = (height x base) ÷ 2
$= (MN \times NO) \div 2$
$= \frac{(\sqrt{5}-1)(\sqrt{15}-\sqrt{3})}{2} = \frac{\sqrt{5^2 \times 3} - \sqrt{15} - \sqrt{15} + \sqrt{3}}{2}$
$= \frac{5\sqrt{3} - 2\sqrt{15}}{2} + \sqrt{3} = \frac{6\sqrt{3} - 2\sqrt{15}}{2}$

P-3