15° & 75° are complementary angle.
since $15^\circ + 75^\circ = 90^\circ$

Thus, $\cos 75^\circ = \sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$.

3a) let $a = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $b = \frac{\sqrt{2+\sqrt{3}}}{2}$.

To compare a & b we square both numbers.

$a^2 = \frac{8+4\sqrt{3}}{16}$ & $b^2 = \frac{2+\sqrt{3}}{4}$ $\qquad 2+\sqrt{3} > 0$.

$a^2 = \frac{2+\sqrt{3}}{4}$

hence, $a^2 = b^2$. (comparing two ⊕ve no.s is the same as comparing their squares).
but a & b are ⊕ve.

Thus, $a = b$.

b) $\tan 75^\circ = \frac{\sin 75^\circ}{\cos 75^\circ}$

$= \dfrac{\frac{\sqrt{2+\sqrt{3}}}{2}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$ but $\frac{\sqrt{2+\sqrt{3}}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$ (proved)

So, $\tan 75^\circ = \dfrac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$= \frac{(\sqrt{6}+\sqrt{2}) \times (\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$

$= \frac{(\sqrt{6}+\sqrt{2})^2}{(\sqrt{6})^2 - \sqrt{2}^2}$

$= \frac{8+4\sqrt{3}}{4}$

Thus, $\tan 75^\circ = 2+\sqrt{3}$.

$\tan 75^\circ \times \cot 75^\circ = 1$

So, $\cot 75^\circ = \frac{1}{\sqrt{3}+2} \times \frac{(\sqrt{3}-2)}{(\sqrt{3})-2)}$

Thus, $\cot 75^\circ = -\sqrt{3}+2$