b_ $RA = (\sqrt{2}-1)(\sqrt{2}+1) - (2\sqrt{2}-3)^2 - 8(\sqrt{2}-2)$

$= \sqrt{2}^2 - (1)^2 - [(2\sqrt{2})^2 - 2(2\sqrt{2})(3) + 3^2] - 8\sqrt{2} + 16$

$= 2 - 1 - [8 - 12\sqrt{2} + 9] - 8\sqrt{2} + 16$

$= 2 - 1 - 8 + 12\sqrt{2} - 9 - 8\sqrt{2} + 16$

$RA = 4\sqrt{2}$ cm

where $a = 4$ which is a natural integer

c_ In Δ LAR we have:

$LA = LR = 4$ cm (proved)

hence, LAR is isosceles Δ at L

Apply converse of pythagorean theorem:

$hyp^2 \overset{?}{=} leg_1^2 + leg_2^2$

$AR^2 \overset{?}{=} LA^2 + LR^2$

$(4\sqrt{2})^2 \overset{?}{=} 4^2 + 4^2$

$32 \overset{?}{=} 16 + 16$

$32 = 32$

hence, LAR is right Δ at L

thus, LAR is right isosceles Δ at L.

2) a_ $RO = \frac{(2\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \frac{4 - 2\sqrt{2} - \sqrt{2} + 1}{2 - 1}$

$RO = (5 - 3\sqrt{2})$ cm

b_ In Δ's ROS & RAL we have:

O & S belong to [RL] & [RA] respectively (given)

so, Apply converse of Thales property: If a st. line divides the sides of a Δ proportionally then it is parallel to the third.

Ratios: If $\frac{RO}{RL} = \frac{RS}{RA} = \frac{OS}{LA}$
(1) (2) (3)