التاريخ:

$E(\sqrt{2}) = 4\sqrt{2} - 6 - 4\sqrt{2} + 5$

Thus, $E(\sqrt{2}) = -1$ which is an integer.

3a) $F(x) = (a+x)^2 - (x^2+1)(b+2x) + a(x^3-a) + c(x-1) - 1$

$= a^2 + 2ax + x^2 - (bx^2 + 2x^3 + b + 2x) + ax^3 - a^2 + cx - c - 1.$

Thus, $F(x) = (a-2)x^3 + (1-b)x^2 + (2a+c-2)x - b - c - 1.$

b) $E(x) \equiv F(x)$ (given).

So,

- coefficients of x^3: $a-2 = 2$

  $\boxed{a=4}$

- coefficients of x^2: $1-b = -3$

  $\boxed{b=4}$

- coefficients of x: $2a+c-2 = -4$

  $8 + c - 2 = -4$

  $\boxed{c=-10}$

- constants: $-b-c-1 \stackrel{?}{=} 5.$

  $-4 + 10 - 1 \stackrel{?}{=} 5$

  $5 = 5$ ✓

4) $G(x) = (4x^2 - 4x + 1) + (6x-3)(x+2) - 5 + 20x^2.$

$= (2x)^2 - 2(2x)(1) + (1)^2 + 3(2x-1)(x+2) + 5(4x^2-1)$

$= (2x-1)^2 + 3(2x-1)(x+2) + 5(2x-1)(2x+1)$

$= (2x-1)[(2x-1) + 3(x+2) + 5(2x+1)]$

$G(x) = (2x-1)(10+15x)$

To find roots of $G(x)$, means solve $G(x) = 0$

$(10+15x)(2x-1) = 0$

means $x = -\frac{2}{3}$ OR $x = \frac{1}{2}$. P-5