

$$3) a) b = \frac{2-\sqrt{3}}{(5-2\sqrt{6})(5+2\sqrt{6})} = \frac{2-\sqrt{3}}{25-24} = 2-\sqrt{3} \checkmark$$

b) if $2-\sqrt{3}$ is a root of $E(x)$ then

$$E(2-\sqrt{3}) = 0 \checkmark$$

$$\begin{aligned} E(2-\sqrt{3}) &= (2-\sqrt{3})^2 - 4(2-\sqrt{3}) + c \\ &= 4 - 4\sqrt{3} + 3 - 8 + 4\sqrt{3} + c \\ &= -1 + c \end{aligned}$$

$$-1 + c = 0 \checkmark$$

$$\boxed{c = 1}$$

$$c) x^2 = (2+\sqrt{3})^2$$

$$= 4 + 4\sqrt{3} + 3$$

$$\boxed{x^2 = 7 + 4\sqrt{3}}$$

$$4x-1 = 4(2+\sqrt{3})-1$$

$$= 8 + 4\sqrt{3} - 1$$

$$\boxed{4x-1 = 7 + 4\sqrt{3}}$$

$$x^2 = 4x-1 \quad (\text{for } x = 2+\sqrt{3}) \checkmark$$

→ the roots of $x^2 - 4x + 1$ are $2-\sqrt{3}$ (given) and $2+\sqrt{3}$ (the difference of 2 equal nbs is zero)

$$\begin{aligned} \text{to prove it: } x^2 - 4x + 1 &= x^2 - 4x + 4 - 3 \\ &= (x-2)^2 - (\sqrt{3})^2 \\ &= (x-2-\sqrt{3})(x-2+\sqrt{3}) \end{aligned}$$

$$\text{So, if } (x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$$

$$\text{then } \underline{x-2-\sqrt{3}} = 0 \quad \text{or} \quad x-2+\sqrt{3} = 0$$

$$x = 2+\sqrt{3} \checkmark$$

$$x = 2-\sqrt{3} \checkmark$$