3a) In the two triangles VAN and KBO:
$\hat{AVN} = \hat{OKB} = 90°$ (proved) and $\underbrace{\hat{NAV} = \frac{\widehat{AV}}{2}}_{\text{angle formed by a tangent and a chord}} = \underbrace{\hat{ABV}}_{\text{inscribed angles}}$

so the 2 Δs VAN and KBO are similar (2 equal respectively angles) (3/4 pt)

b) ratio of similarity $\frac{VAN}{KBO}$: $\frac{VA}{KB} = \frac{AN}{BO} = \frac{VN}{KO}$ but KB = VK
(K is the midpt of [BV]
and VA = 2KO (midpoint theorem in ΔABV) then
$\frac{2KO}{VK} = \frac{VN}{KO}$ so $VN \times VK = 2KO^2$ (3/4 pt)

4) The quadrilateral OANK is formed of two right Δs NAO at A and NKO at K (given and proved) having the same hypotenuse [NO] so the points O, A, N, and K belong to the same circle of diameter [NO] the common hypotenuse (3/4 pt)

5) The circle (C') circumscribed about OANK has a center I the midpoint of [NO] so IO = IA = radii of (C') then I is equidistant from the two fixed points A and O then I varies on the perpendicular bisector of [AO] (3/4 pt).

6) a) $E = t_{\overrightarrow{OK}}^{K}$ so $\overrightarrow{KE} = \overrightarrow{OK}$ so K is the midpoint of [OE] ($\frac{1}{4}$ pt)

b) To prove that $\overrightarrow{AE} = \overrightarrow{AV} + \overrightarrow{AO}$ means that we need to prove that AVEO is a parallelogram.
In ΔABV, $\overrightarrow{AV} = 2\overrightarrow{OK}$ (midpoint theorem) but $\overrightarrow{OK} = \overrightarrow{KE}$ (given)
so $\overrightarrow{AV} = \overrightarrow{OE}$ Therefore AVEO is a parallelogram (2 opposite sides are parallel and equal) hence
$\overrightarrow{AE} = \overrightarrow{AV} + \overrightarrow{AO}$ (E is the 4th vertex of the parallelogram)

Good Work in
Your official
Exams.