

3) In  $\triangle MGF$  we have:

(d) is tangent to (o) at A (given).

So, (MG)  $\perp$  (FA) at A (Tangent theorem: tangent & radius are perp) Rule-1

then, (FA) is a height relative to (MG).

(MF) is tangent to (o) at E (given)

So, (OE)  $\perp$  (MF) at E (Tangent theorem: by a)

but G, O & E are collinear (given)

then, (GE) is a height relative to (MF) at E.

but, (GE) & (FA) intersect at O.

Thus, O is the orthocenter of  $\triangle MGF$ . (intersection pt of 2 heights)

\* O is orthocenter of  $\triangle MGF$  (proved)

So, (MO) is a height relative to (GF) (line issued from vertex & pass through orthocenter)

So, (MO)  $\perp$  (GF)

S is the orthogonal projection of O on (GF) (given)

then, (OS)  $\perp$  (GF) at S.

hence, (MO)  $\parallel$  (OS) (two lines perp. to same st. line are parallel)

but O is a common pt.

Thus, pt M, O & S are collinear.

4) (OE)  $\perp$  (MF) at E (proved)

So,  $\hat{OEF} = 90^\circ$

(OS)  $\perp$  (GF) at S (proved)

So,  $\hat{OSF} = 90^\circ$ .

Thus, quadrilateral EOSF is inscribed in a circle whose center is midpt of (OF) & diameter (OF) (formed of two right  $\angle$ 's sharing same hypotenuse)

5) In  $\triangle MGF$  we have:

(MO) is the bisector of  $\hat{AME}$  (proved)

(MO) is a height relative to (GF) (proved)

Thus,  $\triangle GMF$  is isosceles at M (having height as a bisector).