

### 3<sup>rd</sup> exercise

1)  $f(x) = x^3 + 3 - 3x^2 - x$

-  $f(x)$  is a polynomial then it admits all real no.s as a domain.

OR  $f(x)$  is defined for all real values of  $x$  since  $f(x)$  doesn't have a denominator or a radical that would make it in need of a condition.

Thus,  $f(x)$  admits all real no.s as a domain of definition.

2) a)  $f(x) = x^3 - x + 3 - 3x^2$   
 $= x(x^2 - 1) - 3(x^2 - 1)$   
 $= (x-1)(x+1)(x-3)$

b)  $f(x+1) = (x+1-1)(x+1+1)(x+1-3)$   
 $= (x)(x+2)(x-2)$   
 $= x(x^2 - 4)$

$$\begin{aligned} f(x+1) - f(x-1) &= x(x+2)(x-2) - x(x-2)(x-4) \\ &= x(x-2)(x+2 - x + 4) \\ &= x(x-2)(4x+2) \\ &= 2x(x-2)(2x+1) \checkmark \end{aligned}$$

c)  $f(x+1) = f(x-1)$   
 $x(x+2)(x-2) = x(x-2)(x-4)$   
 $x(x+2)(x-2) - x(x-2)(x-4) = 0$   
 $2x(x-2)(2x+1) = 0$

So,  $x=0$  or  $x=2$  or  $x = -\frac{1}{2}$   $\rightarrow$  rejected since  $-\frac{1}{2}$  is not a natural no.

So, solution set  $\{0; 2\}$  for natural nos.

P.6.