[MP) is tangent to (C) at P (proved)

Then, (MO) is perp. bisector [PQ] (Tangent theorem: Line joining exterior pt. from which tangents are drawn & center of circle is perp. bisector of chord formed by pts of tangencies)

but, O' is (OM) (given)

Thus, (OO') is perp. bisector of [PQ].

b) In $\triangle MPQ$ we have

[MP) and [MQ) are tangents to (C) at P & Q resp. (proved)

So, M is equidistant from P & Q (Tangent theorem: exterior pt from which tangents are drawn is equidistant from pts of tangencies)

So, $PM = QM$.

hence arc $\widehat{PM}$ = arc $\widehat{QM}$ (arcs held by equal chords are equal)

Thus, M is the midpt of arc $\widehat{PQ}$.

4 a) In right triangle OPM of hyp. [OM] we have:

$O\hat{M}Q = 30^\circ$ (given)

Then $\triangle OPM$ is semi equilateral at P.

So, $MP = \frac{\sqrt{3}}{2} \times hyp$ (side facing 60°

$= \frac{\sqrt{3}}{2} \times OM$

but $OM = 2\,O'M$ (radius & diameter of (C'))

$= 12\,cm$

Thus, $MP = \frac{12\sqrt{3}}{2} = 6\sqrt{3}\,cm$

but $MP = QM$ (proved)

Thus, $QM = 6\sqrt{3}\,cm$

b) In $\triangle MPQ$ we have:

$MP = MQ$ (proved)

So, $\triangle MPQ$ is isosceles at M.

(MO) is perp. bisector of [PQ] (proved)

Then, [MO) is the bisector $P\hat{M}Q$ (remarkable line issued from main vertex)

but, $P\hat{M}O = 30^\circ$

then, $P\hat{M}Q = 60^\circ$

hence, $\triangle PQM$ is equilateral.

Thus, $P_{MPQ} = 3$ side

$= 3(PM)$

$= 18\sqrt{3}\,cm$

Rabih S. Khater

P-67