6) a) In $\triangle AME$ we have:

[MA) & [ME) are tangents to (C) at A & E resp (given)

So, $MA = ME$. (Tangent theorem: pt of intersection of tangents is equidistant from pts of tangencies)

& $A\hat{M}E = 60°$ (given)

Rule-i

hence $\triangle AME$ is equilateral (having pair of equal sides + 60°)

So, $Area_{AME} = \frac{Base \times height}{2}$, $height = \frac{\sqrt{3}}{2} \times side$ (height in equi $\triangle$)

$= \frac{AM \times \frac{3R}{2}}{2}$ $\qquad = \frac{\sqrt{3}}{2} AM$

$= \frac{\sqrt{3}}{2}(R\sqrt{3})$

$= \frac{R\sqrt{3} \times 3R}{4}$ $\qquad = \frac{3R}{2}$.

Thus, $Area_{AME} = \frac{3R^2\sqrt{3}}{4}$ units of area.

b) In quadrilateral AOEI we have:

$\triangle AME$ is equilateral (proved).

and [MO) is Bisector of $A\hat{M}E$ (proved)

then, [MO) is perp. bisector of [AE] (Remarkable line in an equi $\triangle$).

but, I belongs to (MO) (given)

So, $IA = IE$ (pt on perp. bisector)

$OA = OE$ (radii of (C)).

In $\triangle AIO$ we have:

$OI = OA$ (radii of (C).

[MO) is bisector of $A\hat{M}E$ (proved)

then, $A\hat{M}O = \frac{A\hat{M}E}{2} = 30°$

but, $M\hat{A}O = 90°$ (proved)

hence, $A\hat{O}I = 60°$ (sum of angles in $\triangle MAO$)

then, $\triangle IOA$ is equilateral (by - i).

then, $AI = AO$ (sides of equilateral $\triangle$)

Thus, AOEI is a rhombus (having 4 equal sides).

Rabih S. Khater
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