Thus, $\triangle ADC$ is right isosceles at D.

$\hat{ACD} = 45°$ (base angle of a right isosceles $\triangle$)

then $\sin \hat{ACD} = \frac{AD}{AC}$

$AC = 2r = a\sqrt{2}$.

$\frac{\sqrt{2}}{2} = \frac{AD}{a\sqrt{2}}$

$\boxed{AD = a \text{ units}}$

① $AD = a$

2). $(LA) \perp (AK)$ (given)

then, $\triangle LAK$ is right of hypotenuse $[LK]$

$(CD) \perp (BC)$ (st. lines held by adj. sides of a square)

then, $\triangle LCK$ is right of hypotenuse $[LK]$

Hence, $\triangle$'s LAK & LCK are two right $\triangle$'s sharing same hypotenuse

Thus, pt A, L, C & K belong to same circle (S) whose center is S the midpt of $[LK]$.

3). In $\triangle$'s ADP & CPK we have:

P belongs to $[DC]$ & $[AK]$ given

And: $(AD) // (KC)$ (two st. lines perp. to same st. line $[DC]$)

Then apply Thale's property:

$$\frac{DP}{PC} = \frac{PA}{PK} = \frac{DA}{KC}.$$

① ② ③

From ratios ① & ③: $\frac{DP}{PC} = \frac{DA}{KC}$.

$\frac{DP}{PC} = \frac{a}{b}$. $(PC = a - DP)$

$\frac{DP}{a - DP} = \frac{a}{b}$

So $b(DP) = a^2 - a(DP)$

$(b+a)(PD) = a^2$

Thus, $PD = \frac{a^2}{a+b}$ unit of length

Rabih. S. Khater.

4) a) In $\triangle$'s ALD & ADP we have:

- $\hat{LAP} = \hat{ADP} = 90°$
- $\hat{APD}$ is a common angle

So, $\triangle$'s are similar by AA-postulate

Ratio: $\triangle \frac{ALP}{DAP}$ | $\frac{AL}{DA} = \frac{AP}{DP} = \frac{LP}{AP}$.

ⓐ ⓑ ⓒ

b) From ratios ⓑ & ⓒ

$\frac{AP}{DP} = \frac{LP}{AP}$

So $AP^2 = LP(DP)$ but $(LP = LD + DP)$

$AP^2 = (LD + DP)(DP)$

$AP^2 = (LD)(DP) + DP^2$

but $AP^2 = AD^2 + DP^2$ (pyth. in $\triangle ADP$)

So $AD^2 + DP^2 = LD(DP) + DP^2$

$AD^2 = LD(DP)$

$LD = \frac{AD^2}{DP} = \frac{a^2}{\frac{a^2}{a+b}} = \boxed{a+b}$ unit