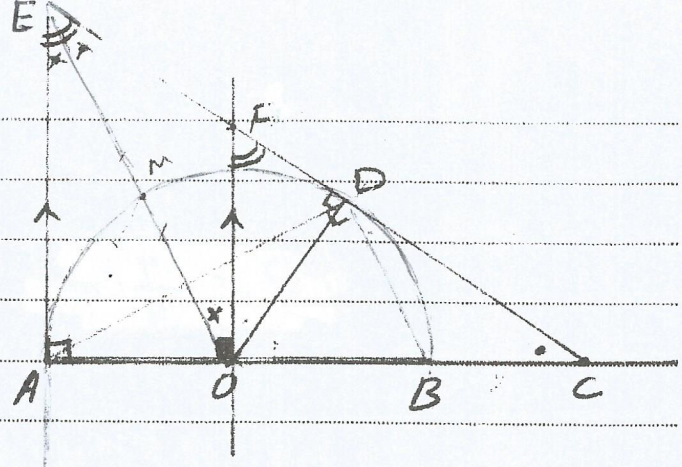
4th exercise: (1pt)

1) figure




2) In Δs ACD + BCD we have

$D\hat{C}B = D\hat{C}A$ (common angle)

(CD) is tangent to (C) at D (given)

So, $B\hat{D}C = \frac{\text{mes } \overset{\frown}{BD}}{2}$ (angle formed by tangent + a chord)

$D\hat{A}C = \frac{\text{mes } \overset{\frown}{BD}}{2}$ (inscribed angle intercepting arc BD)

hence, $B\hat{D}C = D\hat{A}C$ (by comparison) (1pt)

Thus, Δs DCA + BCD are similar by A.A postulate.

Ratio of similitude: $\frac{DACD}{DCB} \Big| \frac{AC}{DC} = \frac{AD}{DB} = \frac{CD}{CB} = \text{cste}$

3 a) In Δ ODC we have

(CD) is tangent to (C) at D (given)

+ [OD] is a radius of C at O (1) (1½pts)

So, $O\hat{D}C = 90°$ (1) Tangent Theorem: angle formed by a tangent + a radius)

So $\cos O\hat{C}D = \frac{\text{adj}}{\text{hyp}} = \frac{CD}{OC} = \frac{CD}{5}$.

In Δ OFC we have

(FO) ⊥ (OC) (given).

So, $\cos O\hat{C}D = \frac{\text{adj}}{\text{hyp}} = \frac{OC}{CF} = \frac{5}{CF}$

So, $\frac{CD}{5} = \frac{5}{CF}$

$CD \times CF = 25$

$CF = \frac{25}{4}$ cm but C, D + F are ... collinear

In right Δ OCD apply pyth. Theorem

$OC^2 = OD^2 + DC^2$

$DC^2 = 25 - 9$

$DC = 4\text{cm}$