

St. line joining exterior pt from which two tangents are drawn to center of circle is the bisector of the exterior angle formed by the two tangents

b) In $\triangle DPB$ we have:

$$\hat{D}B O = 90^\circ \text{ (proved)}$$

$$\hat{O}D B = 30^\circ \text{ (sum of angles in } \triangle D O B \text{)}$$

but $[DO]$ is the bisector of $\hat{B}D E$ (proved)

$$\begin{aligned} \text{Then, } \hat{B}D E &= 2 \hat{O}D B \\ &= 60^\circ. \end{aligned}$$

Hence, $\triangle DBP$ is semi-equilateral at B . (having $60^\circ + 90^\circ$)

Thus, $PD = 2$ smallest side (hyp of a semi-equilateral)

$$PD = 2DB.$$

c) In $\triangle DPO$ we have:

$$\hat{B}D E = 60^\circ \text{ (proved)}$$

$$\hat{O}D B = 30^\circ \text{ (proved)}$$

\rightarrow then, $\hat{O}D E = 30^\circ$.

$\triangle DPB$ is semi-equilateral at B with $\hat{B}D P = 60^\circ$ (proved)

\rightarrow So, $\hat{O}P E = 30^\circ$

hence $\triangle OPD$ is isosceles at O (having two equal angles)

but $[DE]$ is tangent to (C) at E (given)

So, $(OE) \perp (PD)$ (Tangent Theorem: tangent and radius are \perp)

then, (OE) is a height issued from main vertex of iso $\triangle OPD$.

hence, (OE) is a median relative to $[PD]$ at E .

Thus, E is the midpt of $[PB]$.