

\* In  $\Delta HGF$  we have:

-  $[OS]$  is perp. to  $[GF]$  (given)

-  $O$  is the orthocenter of  $\Delta HGF$  (proved)

-  $H$  is a vertex of  $\Delta HGF$  facing  $[GF]$  (given)

then,  $[HS]$  is a height relative to  $[GF]$

thus,  $H, O, S$  are collinear.

4. a) \* In  $\Delta OEF$  we have:

$[OE]$  is perp. to  $[EF]$  (proved)

then,  $OEF$  is right  $\Delta$  of hyp.  $[OF]$ .

\* In  $\Delta OSF$  we have:

$[OS]$  is perp. to  $[SF]$  (proved)

then,  $OSF$  is right  $\Delta$  of hyp.  $[OF]$ .

\* In quad.  $OESF$  we have:

$OEF$  &  $OSF$  are right  $\Delta$ s of same hypotenuse  $[OF]$  (proved)

thus,  $O, E, F, S$  belong to the same circle of center  $K$ , the midpt. of  $[OF]$ . (quad. formed of 2 right  $\Delta$ s sharing same hyp.)

5.  $[MO]$  is the perp. bisector of  $[AE]$  (proved)

$I$  is the midpt. of  $[AE]$  (given)

then,  $\hat{AIO} = 90^\circ$

so,  $AIO$  is right  $\Delta$  at  $I$

but  $A$  &  $O$  are fixed pts. (given)

thus, as  $M$  varies on  $(O)$ ,  $I$  will describe a circle of center, midpt. of  $[AO]$ , and diameter  $[AO]$ .