

$$\begin{aligned}
 3) \text{ Area of } (c) &= \pi r^2 \\
 &= \pi (OA)^2 \\
 &= \pi \left(5\frac{\sqrt{2}}{2}\right)^2 \\
 &= \frac{25\pi}{2} \text{ cm}^2
 \end{aligned}$$

$r =$ radius of (c)
 $AB = 5 \text{ cm}$ (given)
 $OA = \frac{\sqrt{2}}{2} \times \text{hyp}$ (leg of a right \triangle)
 $OA = \frac{\sqrt{2}}{2} \times (AB)$
 $= \frac{5\sqrt{2}}{2} \text{ cm}$

4) a) Done ✓

b) H is the orth. proj of I on (OA) (given)

then, $(IH) \perp (OA)$

$\triangle AOB$ is right at O . (given)

So, $(AO) \perp (OB)$

Thus, (IH) is parallel (OB) (Two st. lines perp to same st. line in plane are parallel).

5) In $\triangle AHI$ & $\triangle AOB$ sharing same vertex A . we have:

$(HI) \parallel (OB)$ (proved)

AH & AO are collinear

A, I & B are collinear in this order

Then, use Thales' property: (If a st. line is drawn parallel to a side of a \triangle then it cuts other sides proportionally)

Thales' ratios:

$$\frac{AI}{AB} = \frac{AH}{AO} = \frac{IH}{BO} = \text{cst}$$

Using ratios (1) & (2)

$$\frac{AI}{AB} = \frac{AH}{AO}$$

$$\frac{1}{5} = \frac{AH}{5\frac{\sqrt{2}}{2}}$$

$$\text{Thus, } \boxed{AH = \frac{\sqrt{2}}{2} \text{ cm}}$$

Using (1) & (3)

$$\frac{1}{5} = \frac{IH}{5\frac{\sqrt{2}}{2}}$$

$$\text{Thus, } \boxed{IH = \frac{\sqrt{2}}{2} \text{ cm}}$$

6) $(HI) \perp (AO)$ proved.

So, $\angle IHO = 90^\circ$.

F is diametrically opp. to A (given)

Then, B is on (c) of diameter $[AF]$

Then, $\angle IBF = 90^\circ$.

hence, sum of opposite angles in quad. $HIBF$ is 180° .

Thus, pts H, I, B & F belong to same circle of diameter $[IF]$.