

$$3a) \hat{BAC} = \frac{1}{2} \hat{BOC} \text{ (inscribed + central angle intercepting arc BC)}$$

$$= 50^\circ$$

$$\hat{COA} = 80^\circ \text{ (proved)}$$

(OD) is bisector of \hat{COA} (given)

$$\text{Thus, } \hat{DOA} = \frac{\hat{COA}}{2}$$

$$= 40^\circ$$

b) In quadrilateral $CDOI$ we have:

C is a pt on (c) of diameter $[AB]$ (given)

then, $\hat{ACB} = 90^\circ$ (inscribed angle facing diameter)

but $\hat{BAC} = 50^\circ$ (proved)

So, $\hat{ABC} = 40^\circ$ (sum of base angles in a right Δ)

but $OB = OC$ (radii of (c))

then ΔOBC is isosceles at O (having two equal sides)

but $\hat{BOC} = 100^\circ$ (given)

+ base angles in an isosceles Δ are equal

then $\hat{OCB} = \frac{180 - \hat{COB}}{2}$ (sum of angles in a Δ)

$$\hat{OCB} = 40^\circ$$

then, $\hat{OCB} = \hat{COD} = 40^\circ$ (by comparison)

hence $(OD) \parallel (CB)$ (line included between equal alt.

but I belongs to (CB) interior angles)

so, $(OD) \parallel (OI)$

+ $(CD) \parallel (OI)$ given.

Thus, $CDOI$ is a parallelogram (having 2 pairs of \parallel sides).

c) $CDOI$ is a parallelogram (proved)

$$\text{Then, } \vec{OI} = \vec{DC}$$

Thus, E is the image of D by \vec{OI} .

d) $CDOI$ is a parm (proved) but $(OP) \perp (EF)$ given

d) So, P is midpt of diagonal $[DI]$ then, (OP) is a median relative to $[EF]$.

$$OE = OF \text{ (radii of } (c))$$

hence, P is midpt of diagonal $[EF]$

So, $OEFI$ is isosceles at O .

Thus, $DEFI$ is parm.