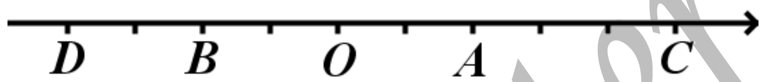
Al Mahdi High Schools (Al-Hadath) — Mathematics — 10^{th}-Grade

Name: . . . . . . **" Absolute Value & Intervals"** — S.S-1.2

A- Introduction:

Consider the following number line:



1- Find the abscissas of the above points.

……………………………………………………………………………………

2- Determine the distances between the given points and origin.

……………………………………………………………………………………

3- Denote by $|x_A|$ & $|x_B|$ (read absolute values of x_A & x_B) the distances $OA \& OB$ respectively then Compare:

  a. Abscissas of the points ***A*** & ***B***.

  ……………………………………………………………………

  b. Distances of the points ***A*** & ***B*** from the origin ***O***.

  ……………………………………………………………………

4- What do you conclude?

……………………………………………………………………………………

……………………………………………………………………………………

5- What does $|x_D|$ mean? ……………………………………………………………

6- Compare: $|x_D|$ & x_D. ……………………………………………………………

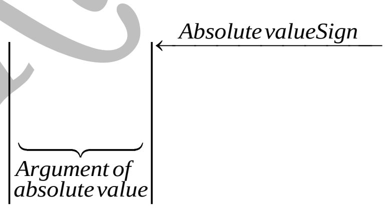
7- What do you conclude? ……………………………………………………………

B- Definition and Terminology of Absolute value:

i- *Geometric definition of $|x|$:*

We define the absolute value of a number as the ***distance*** of a number ***from zero***.

TERMINOLOGY:



ii- *Algebraic definition of $|x|$:*

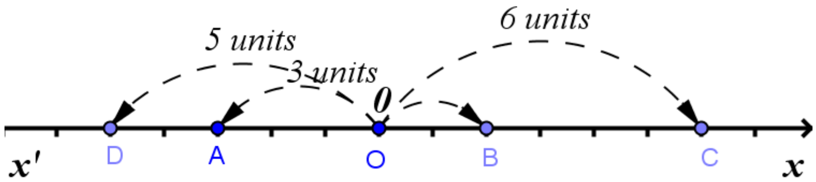
$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases} \quad \text{Meaning the answer is} \begin{cases} x \text{ itself if } x \text{ is } \textbf{positive} \\ \textbf{opposite} \text{ of } x \text{ if } x \text{ is } \textbf{negative} \\ \text{zero if } x \text{ is } \textbf{zero}. \end{cases}$$

Ex_1 : *Write the following without absolute value sign:*

$a)\ |2| = ...$ $b)\ |-3| = ...$

$c)\ |\sqrt{5} - 2| = ...$ $d)\ |\sqrt{2} - 3| = ...$

Again, let us consider the following number line



As you noticed from the above axis that the distance: $OA = 3units$, $OC = 6units$ & $OD = 5units$

Whereas the abscissas of: $A(-3), C(+6)$ & $D(-5)$.

Conclusions:

Fill in the blanks with most suitable words: (*never negative, can be negative, how far, where to*)

1- Since $|x - 0|$ represents distance of a point of abscissa x from origin and distance is never negative then absolute value is ……………………………………………………………….....
2- Since distance asks how far then absolute value also asks ………………………………...

LOOK **ABSOLUTE VALUE ONLY ASKS**

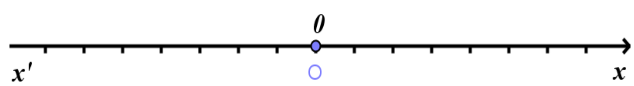
How far?

Not in Which direction?

This means that not only $|3| = 3$ but, also that $|-3| = 3$ remember absolute value measures ***distance***.

Ex$_2$: Consider the points M, N & P so that $OM = 1cm, ON = 3cm$ & $OP = 5cm$.

a) What does ON represent? ……………………..



b) Detect the possible positions of the above points.

c) Find the values of x the abscissa of:

i. M if, $OM = |x|$ ………………………………………………………………
ii. P if, $OP = |x|$ ………………………………………………………………

It is important to note that the ***absolute value*** bars do ***NOT*** work in the same way as ***parentheses*** do.

Ex$_3$: **Simplify**

$-(-5) = +5$, this is NOT how it works for absolute value:

$-|-5| = -(5) \leftarrow$ First handle inside the absolute value part

$= -5 \leftarrow$ Then workout the parentheses and outside absolute value

No.	Simplify	Your answer
1.	$-2\times\|2-5\|$	
2.	$-2\times\|2+3(4-5)\|$	
3.	$2-\|3-5\|^2$	

C- *Properties of absolute values*

Prop.	*Algebraically*		*In words*
1.	$\lvert x\rvert = \lvert -x\rvert$		A number and its opposite in absolute value are ***equal***.
2.	$x \le \lvert x\rvert$		A number is always ***less than or equal*** to its absolute value
3.	$\lvert x\rvert = \lvert y\rvert \Leftrightarrow \begin{cases} x = y \\ or \\ x = -y \end{cases}$		Two numbers in absolute values are equal is means that these numbers are ***equal*** or a number is ***equal to the opposite*** of the other and vice versa.
4.	$\lvert x \times y\rvert = \lvert x\rvert \times \lvert y\rvert$		The product of two numbers in absolute is equal to the product of their absolutes
5.	$\left\lvert \frac{x}{y} \right\rvert = \frac{\lvert x\rvert}{\lvert y\rvert}$ s.t $y \ne 0.$		The quotient of two numbers in absolute is equal to the quotient of their absolutes so that the denominator is different than zero.
6.	a.	$\lvert x + y\rvert \le \lvert x\rvert + \lvert y\rvert$	***Triangular inequalities***
	b.	$\lvert x - y\rvert \le \lvert x\rvert + \lvert y\rvert.$	
	c.	$\lvert x + y\rvert \ge \lvert x\rvert - \lvert y\rvert$	
7.	$x^2 = \lvert x^2\rvert = \lvert x\rvert^2.$		A number squared is equal to its square in absolute value is equal to its absolute value squared.
8.	$\sqrt{x^2} = \lvert x\rvert$		The square root of number is equal to its absolute value.

Discuss the following:

Part	Statement	Short solution with justification
1.	Compare $\lvert 2 - x\rvert$ & $\lvert x - 2\rvert$	
2.	Solve $\lvert x\rvert = -3$	
3.	Interpret $\lvert x\rvert = 3$	
4.	Interpret $\lvert x - 5\rvert = 7$	

D- Absolute values and inequalities:

If r is any real number that belongs to the set of real numbers $\mathbb{R}$ then we can write:

The absolute values	In interval form as	In double inequality form as
$\|x\| \le r$	$x \in [-r;+r]$	$-r \le x \le +r$
$\|x\| < r$	$x \in]-r;+r[$	$-r < x < +r$
$\|x\| \ge r$	$x \in]-\infty;-r] \cup [+r;+\infty[$	$x \ge +r \;\; OR \;\; x \le -r$
$\|x\| > r$	$x \in]-\infty;-r[\cup]+r;+\infty[$	$x > +r \;\; OR \;\; x < -r$
$\|x-a\| \le r$	$x \in [a-r;a+r]$	$a-r \le x \le a+r$
$\|x-a\| \ge r$	$x \in]-\infty;a-r] \cup [a+r;+\infty[$	$x \ge a+r \;\; OR \;\; x \le a-r$
$\|x-a\| > r$	$x \in]-\infty;a-r[\cup]a+r;+\infty[$	$x > a+r \;\; OR \;\; x < a-r$

Absolute values	Meaning in words	On number line
$\|x\| \le r$	The distance between a point $M(x)$ and $O(0)$ is less than or equal r	
$\|x-a\| > r$	The distance between a point $M(x)$ and $N(a)$ is strictly greater than r	
$\|x-a\| \le r$	The distance between a point $M(x)$ and $N(a)$ is less than or equal r	

E- Important examples:

$|x| \le -5$ is impossible for all $x \in \mathbb{R}$

$|x| \ge -3$ is true for all $x \in \mathbb{R}$

$|x| \le 7$ *means* $x \in [-7;+7]$ *or* $-7 \le x \le +7$.

$x \ge 9$ *means* $x \in]-\infty;-9] \cup [+9;+\infty[$ *or* $(x \le -9 \,\&\, x \ge 9)$