A- Comparing a real number with its square:

How to compare a real number "x" with its square?
Complete the following table:

Steps	*If statement*	*Example*	*Then compare*
1	$x>1$	If $x=3$, so $x^2=.....$	$x^2.....\ x$
2	$x<0$	If $x=-2$, so $x^2=....$	$x^2.....\ x$
3	$0<x<1$	If $x=\frac{1}{5}$, so $x^2=....$	$x^2.....\ x$

Exercise: Compare the following real numbers with their squares:

1) $\pi-3$: ……………………………………………………………………………..
2) $\sqrt{2}-1$: …………………………………………………………………………

B- Comparing two real numbers:

To compare two real numbers "x & y" you have two main ways:

1) By subtraction. $x-y$
2) By comparing their squares. x^2 & y^2
3) By squaring then subtracting the given real numbers or vice versa.

Exercise: Consider the real numbers: $a=\sqrt{5+2\sqrt{5}}$ *and* $b=\sqrt{5-2\sqrt{5}}$.

1) Compare a & b .

……………………………………………………………………………………..
……………………………………………………………………………………..

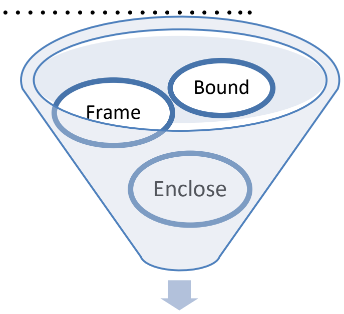
2) Deduce the sign of a^2-b^2.

……………………………………………………………………………………..
……………………………………………………………………………………..

C- Framing real numbers:

To frame a number we can only perform the following operations:

- Multiplication.
- Addition.
- Reciprocal.



all means to encircle a number between two values.

Exercise-1: If $1 < x < 3$, then frame:

1) x^2 ……………………………….. 4) $\frac{1}{x}$ ………………………………..

2) $2x+1$ ……………………………….. 5) $\sqrt{x}-2$ ………………………………..

3) $\sqrt{x}$ ……………………………….. 6) $\frac{\sqrt{x}-2}{x}$ ………………………………..

Take any number a and then complete the following table:

Condition		$a > 1$	$0 < a < 1$
Choose a value for a			
Find its	Reciprocal		
	Square		
	Square root		
Compare the above numbers			

Conclusions:

$$\text{If } a > 1 \text{ then } \frac{1}{a} < 1 < \sqrt{a} < a < a^2.$$

$$\text{If } 0 < a < 1 \text{ then } a^2 < a < \sqrt{a} < 1 < \frac{1}{a}.$$

Perform the following questions on your copy book:

Exercise-2: In each case, give a framing of: $x+y, x-y$ & $2x-3y$

a. $2 \le x \le 5$ & $4 \le y \le 8$.

b. $-5 \le x \le -2$ & $4 \le y \le 8$.

c. $-3 < x < -1$ & $-5 < y < -2$.

Exercise-3: Given that: $-1 \le x \le 3$ & $0 < y < 1$.

Encircle $x - y$ *and* $y - x$.

Def	The framing of the product of xy is obtained by multiplying side by side the two inequalities, this is true *iff* all terms are ***positive***, if not then determine the +ve framing.

Exercise-4: Frame xy & $\frac{x}{y}$ in each of the following cases:

a. $2 \le x \le 5$ & $4 \le y \le 8$.

b. $-5 \le x \le -2$ & $4 \le y \le 8$.

c. $-3 < x < -1$ & $-5 < y < -2$.

Exercise-5: Prove that if $-1 < s \le 2$ and $3 < k \le 4$, then $-4 < sk \le 8$

Exercise-6: Given that: $1 \le a \le 2$ & $2 \le b \le 3$.

Enclose $\frac{1}{b}$ *then* $\frac{a}{b}$.

Exercise-7: Given that: $\frac{2}{3} < r < \frac{3}{2}$ & $-2 < n < -1$.

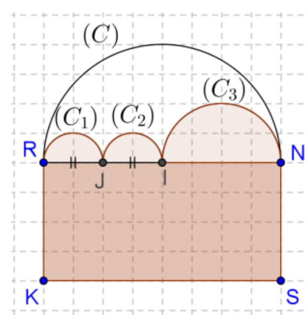
Bound: 1) $-n$
2) $-rn$
3) rn

Exercise-8: observe the framing of x^2 in each of the following cases, then complete the conclusion:

No.	Framing of x	Framing of x^2
1	$2 < x < 3$	$4 < x^2 < 9$
2	$-4 < x < -3$	$9 < x^2 < 16$
3	$-2 < x < 3$	$0 < x^2 < 9$
4	$-5 < x < 3$	$0 < x^2 < 25$

Conclusions	If $a \le x \le b$ where 1) Both a & b are +ve, then $.... \le x^2 \le ....$ 2) Both a & b are - ve, then $.... \le x^2 \le ....$ 3) a & b are of opposite signs and i) $\lvert a \rvert < b$, then $.... \le x^2 \le ....$ ii) …….., then $0 \le x^2 \le a^2$

Exercise-9: In the adjacent figure $RNSK$ is a rectangle of dimensions a & b, where I is the midpoint of $[RN]$, J is the midpoint of $[RI]$ and $1 < a < 2$ & $3 < b < 4$



1) Bound:
   a. The area of $RNSK$
   b. The perimeter of $RNSK$
2) Find the radius of each of the given semi-circles in terms of a
3) Prove that the area of the un-shaded circular region is: $\frac{4\pi}{64} b^2$
4) Can the area of this circular region be $\frac{11\pi}{64}$? Justify.