❑ (Increasing-Decreasing)

A function f defined over an interval $I \subset \Re$ is said to be	
Increasing	Strictly Increasing
If *for all* x_1 & $x_2 \in I$, *such that* $x_1 < x_2$, ***then***	
$f(x_1) \leq f(x_2)$.	$f(x_1) < f(x_2)$.

Ex$_1$: Consider the function $f: y = x^2$.

a) Determine the domain of f . ……………………………………

b) Determine the coordinates of the points $A \& B$.

……………………………………………………………

c) Compare the abscissas of $A \& B$. ……………………………….

d) Compare the ordinates of $A \& B$. ……………………………….

e) What can you say about the variation of C_f ?

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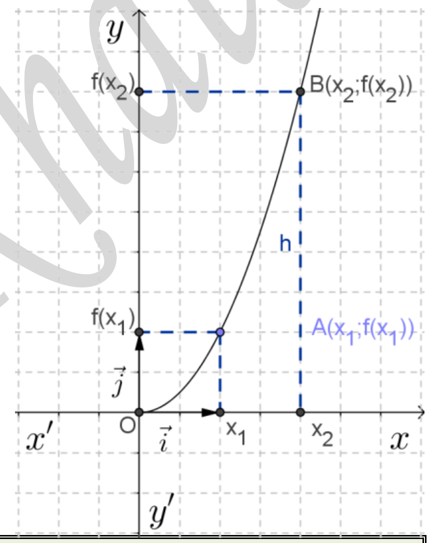
f) What do you conclude?

……………………………………………………………



A function f defined over an interval $I \subset \Re$ is said to be	
Decreasing	Strictly Decreasing
If *for all* x_1 & $x_2 \in I$, *such that* $x_1 < x_2$, ***then***	
$f(x_1) \geq f(x_2)$.	$f(x_1) > f(x_2)$.

Ex$_2$: Is the function $f: y = x^2$ decreasing over $]-\infty; 0[$? Justify.

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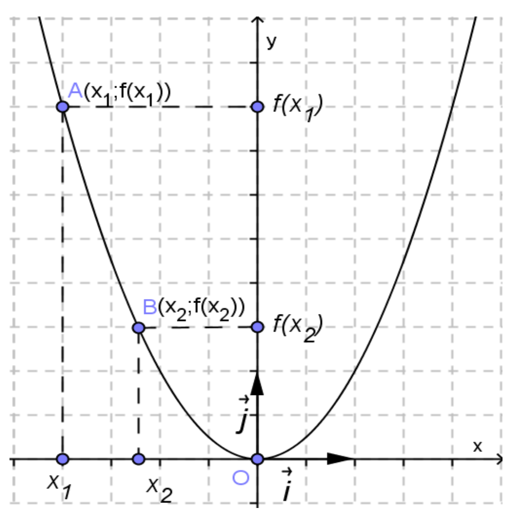

Ex$_3$: Is the function $f: y = -x$ decreasing on $\Re$? Justify.

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❑ Extrema (Maximum- Minimum)

❖ Finding extremum: there are four main ways to find the extremum of a function:

Graphically:

1- Pick up from C_f :

  a. The highest point: ……………………………..

  b. The point with the highest ordinate: …………..

2- Find the maximum height of C_f . ……………………………..

3- For what value of x is this maximum height attained? ……...

4- Which point would be suitable to indicate the maximum of

Using completing the square:

Ex: Consider the expression $f(x) = 2x - 1$ and the inequality $f(x) < 3$.

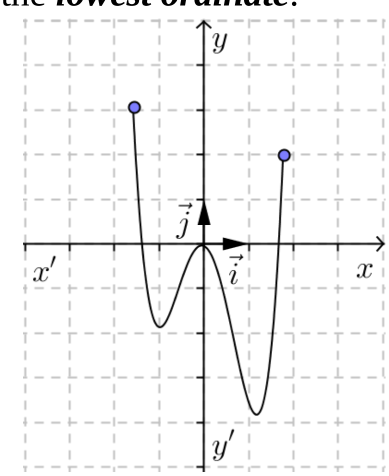
1) What does the above inequality till you? ………………………………………………

2) Solve in set of real numbers: $f(x) < 3$, ………………………………..

3) What is

		Analytically	*Graphically*
Absolute	***Maximum*** of a function f	Is a point on its graph with the ***highest ordinate***.	
	Minimum of a function f	Is a point on its graph with the ***lowest ordinate***.	
Local	***Maximum*** of a function f		
	Minimum of a function f		

- ***Absolute maximum*** of a function f : is a point on its graph with the ***highest ordinate***.
- ***Absolute minimum*** of a function f : is a point on its graph with the ***lowest ordinate***.



Consider the function f defined by $y = f(x)$ & its graph C_f .

1- Find the domain of f . D_f = ...

2- Find the interval (s), for which f is increasing or decreasing.

f is increasing over the interval: . . . .

f is decreasing over the interval: . . . .

3- Find the absolute maximum of f .

f has a maximum: $y =$.........................

and it is reached at $x =$..........................

4- Find the absolute minimum of f .

f has a minimum: $y =$.................... and it is reached at $x =$..........................

5- $(x; f(x))$ represents the coordinates of any point on the graph of f .

a. Calculate the following values: $f(-1)=$........... $f(-1.5)=$.............

$f(0)=$........... $f(2)=$.............

b. Complete: $f(x)\le 3$ for all $x\in D_f$, so $y=3$ *is* ....

$f(x)\ge -3.8$ for all $x\in D_f$, so $y=-3.8$ *is* ....

6- Determine the **local minimum** of f (***minimum over a part of the domain***) over $I=[-1.5;0]$.

f admits a .... at $y=$...................., & it is reached at $x=$................

7- Determine the **local maximum** of f (***maximum over a part of the given domain***) over $I_1=[0;2[$.

f admits a local maximum over the interval I_1 at $y=$.................., & it is reached at $x=$................

8- Set-up the table of variation of f, by completing the below table:

x	-1.5	-1	0	1.1	2
$f(x)$					

Finding extremum: there are three main ways to find extremum of a quadratic function:

$$f(x)=ax^2+bx+c$$

1- *Using completing the square:*

Ex: write the quadratic function: $f(x)=x^2-4x+1$ in form of $f(x)=(x-a)^2+b$ where a & b belong to *R*.

2- *Directly use the formula:*

The abscissa of the extremum is: $x=-\frac{b}{2a}$

The extremum is $f\left(-\frac{b}{2a}\right)$

Ex: if $f(x)=x^2-6x+9$

Then , $\left.\begin{matrix}a=1\\ b=-6\end{matrix}\right\}so, x=-\frac{b}{2a}=\frac{6}{2}=3$

And $f(3)=9-18+2=-7$

3- Show that $\frac{1}{8}$ is a maximum for $f:x\longrightarrow\frac{x-2}{x^2}$

Soln: $f(x)-\frac{1}{8}=\frac{x-2}{x^2}-\frac{1}{8}=-\frac{(x-4)^2}{8x^2}\le 0$

Then $f(x)-\frac{1}{8}\le 0$

Thus, $f(x)\le\frac{1}{8}$ which means that $\frac{1}{8}$ is a maximum for f