

Draw on your copy book the following solids:



It is a bit difficult to represent solid objects of space in a plane.

To facilitate such a task we will use a drawing technique known as cavalier perspective.

❖ **Def:** Perspective (view point) is a drawing technique, where a three dimensional object is represented in a plane.

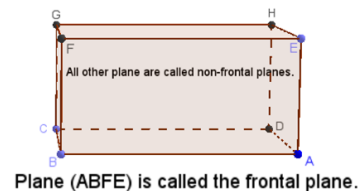
❖ **Types:** There are two main types of perspective:

<i>True perspective, where an escape point exists</i>	<i>Cavalier perspective, where there is no escape point</i>
<p>Left Vanishing Point      Right Vanishing Point</p>	

❖ **Terminologies:**

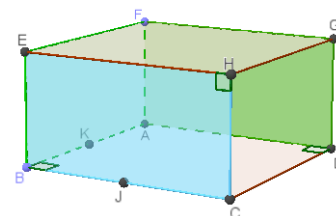
- ❖ A plane is an unlimited surface.
- ❖ Frontal plane: is the plane that is directly in front of observer.
- ❖ Face: is a subset of a plane.
- ❖ Perspective:

- a) Line: is a line that is perpendicular to frontal plane. Ex: (BC) & (AD).
- b) Angle: angle formed by a horizontal line & a perspective line.

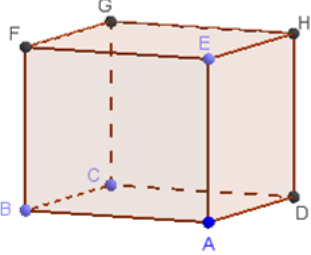
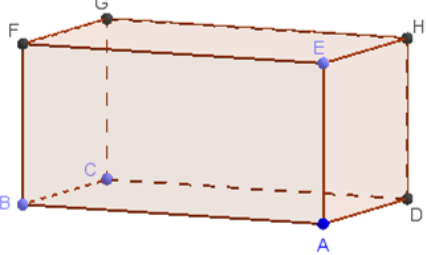
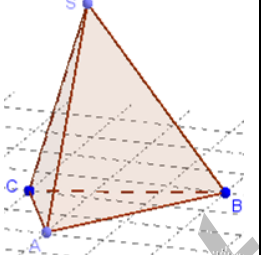
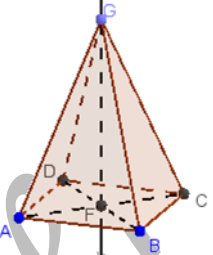


❖ **Principle rules of Cavalier's perspective:**

- 1- The full lines are seen by the observer.
- 2- The dotted lines are hidden with respect to the observer
- 3- Parallelism is conserved, that is parallel lines are presented by parallel straight lines.
- 4- Midpoints of segments are conserved.
- 5- Right angles are represented by right angles only in **frontal** plane.
- 6- Segments subset, of frontal plane are presented in true dimensions.
- 7- The ratio of segments having the same direction is preserved.

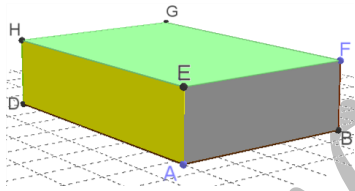


## Basic solids in space

	Cube	Parallelepiped (rectangular prism)	Tetrahedron	Pyramid
<b>Number of:</b>				
<b>Faces</b>				
<b>Edges</b>				
<b>Vertices</b>				

### App-1:

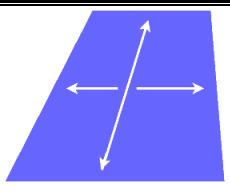
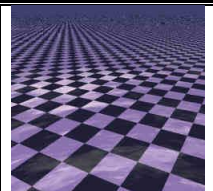
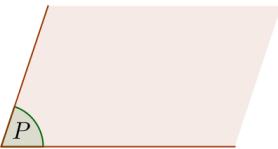
1- Redraw the solid rectangular prism  $ABCDEFGH$  in Cavalier perspective:



2- Complete the following table:

Non-hidden	Edges	
	Faces	
Hidden	Edges	
	Faces	

### Notion about planes

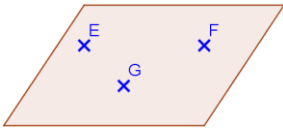
<i>Definition</i>	<i>Properties</i>	
<p>In solid geometry, a plane is a two-dimensional "surface" (similar to the surface of still water, or a sheet of paper, but with no thickness)</p>		
<b>Representation</b>	<p>A plane is generally presented by a parallelogram</p> 	

## Determination of a plane

A unique plane can be determined by:

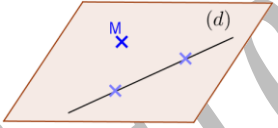
1- Three distinct non-collinear points.

Denoted by  
 $(EFG)$



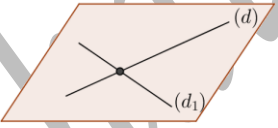
2- A straight line and any point that is not on this straight line.

$pl(M;(d))$



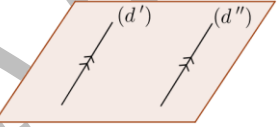
3- Two intersecting straight lines

$pl((d);(d_1))$



4- Two parallel straight lines.

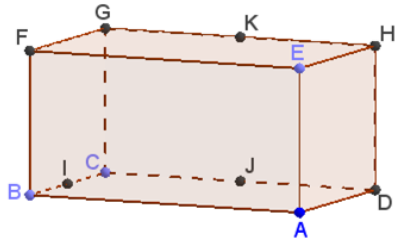
$pl((d');(d''))$



### App-2:

For the given rectangular prism  $ABCDEFGH$  list (more than 2):

1- Straight lines	a) Parallel to $(CG)$	
	b) Subset of plane $(ABC)$	
2- Planes formed of	- Three non collinear points	
	- Two intersecting lines.	
	- $(IK)$ and a point.	
	- Any two parallel st. lines.	

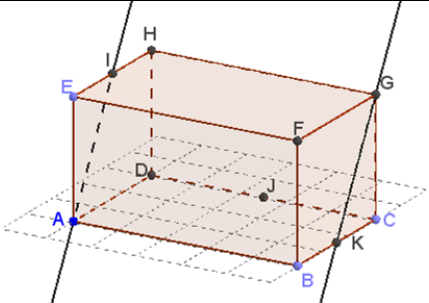
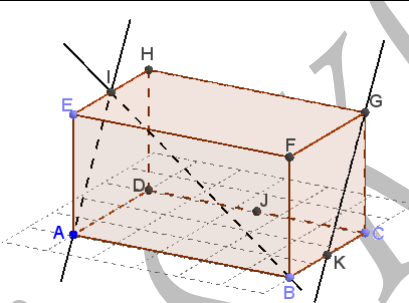
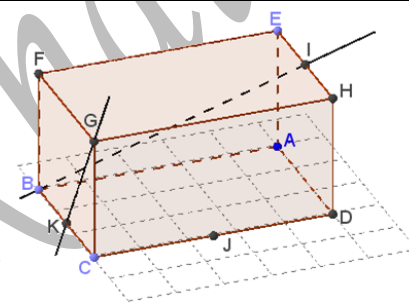


How many planes can be formed of the straight lines  $(KJ)$  &  $(BF)$ ? .....

What can you say about the planes formed in- 2)? .....

## Relative positions of two straight lines in space

In space two straight lines can be:

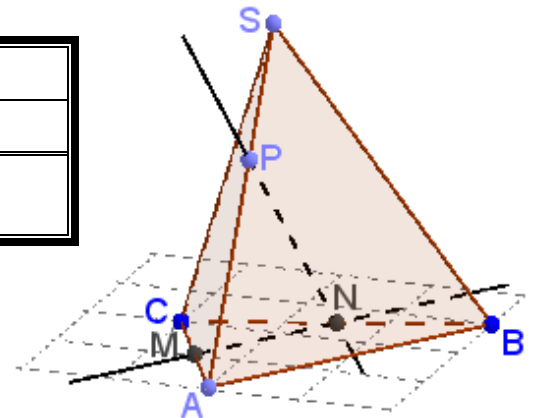
	Coplanar	Non - Coplanar
Definition	Two straight lines are coplanar iff they are subset of the same plane	
Types	Parallel	Intersecting
Graphs		
Analytical study	$\left. \begin{array}{l} (AI) \subset (AKI) \\ (KG) \subset (AKI) \end{array} \right\}$ <p>So, <math>(AI) \&amp; (KG)</math> are coplanar</p> $(AI) \cap (KG) = \emptyset$ <p>Thus, <math>(AI) \parallel (KG)</math></p>	$\left. \begin{array}{l} (AI) \subset (ABI) \\ (BI) \subset (ABI) \end{array} \right\}$ <p>So, <math>(AI) \&amp; (BI)</math> are coplanar</p> $(AI) \cap (BI) = \{I\}$ <p>Thus, <math>(AI) \&amp; (BI)</math> are intersecting</p>
		Skew 
		$\left. \begin{array}{l} (GK) \subset (BCG) \\ (BI) \subset (BDI) \end{array} \right\}$ <p>Thus, <math>(GK) \&amp; (BI)</math> are non - coplanar or skew.</p>

### App-3:

Consider the tetrahedron  $SABC$  where  $M$  &  $N$  are respective midpoints of  $[AC]$  &  $[BC]$  &  $P \in (AS)$ .

1) Indicate the straight lines that are:

Coplanar	Parallel to $(MN)$	
	Intersecting with $(PN)$	
Non-coplanar	$(MP)$	



2) Are the straight lines  $(PN) \& (CS)$  intersecting? Justify.

3) Find 
$$\left\{ \begin{array}{l} (MNP) \cap (ABC) \\ (MNP) \cap (ACS) \\ (MNP) \cap (ABS) \end{array} \right.$$

Relative positions of a straight line and a plane in space

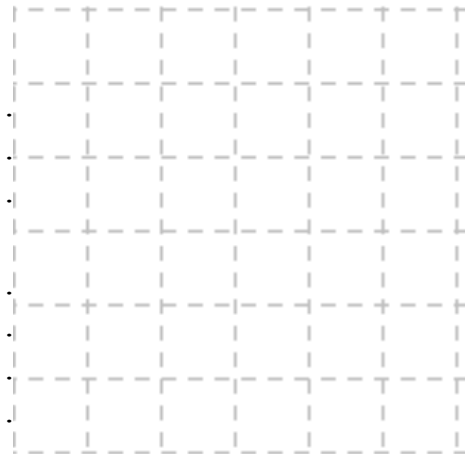
In space a straight line and a plane can be:

Position	<i>Intersecting</i>	<i>Parallel</i>	<i>Subset</i>	<i>Perpendicular</i>
<b>Geometric approach</b>				
<b>Analytic approach</b>	$(Q) \cap (d) = \{G\}$	$(R) \cap (d) = \emptyset$	$(P) \cap (d) = (RN)$	$(d) \perp \left\{ \begin{array}{l} (AB) \& (CD) \\ \& (AB) \cap (CD) \neq \emptyset \end{array} \right.$
<b>Proof?</b>	Find a unique common point	Prove that $(d)$ is parallel to a straight line subset of the plane $(R)$	Find two common points	Prove that $(d)$ is orthogonal to intersecting lines in $(R)$

**App-3:**

Consider the regular tetrahedron  $SABC$ , where  $M$  is the midpoint of  $[AB]$ .

- 1) Trace the figure.
- 2) Prove that  $[CM)$  is perpendicular to  $(AB)$
- 3) Deduce that  $(AB)$  is perpendicular to the plane  $(SMC)$



Relative positions between two planes in space

Position	<i>Parallel</i>	<i>Confounded</i>	<i>Secant(intersecting)</i>
<b>Geometric approach</b>			