Mathematics

To vectors

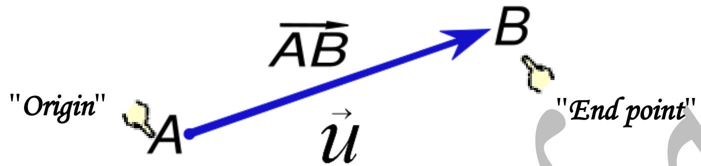
Name: . . . .

10th-Grade

S.S-2.

Definition of a vector:

A vector $\vec{u}$ or $\vec{AB}$ is an ***oriented*** segment having two ***extremities***, an ***origin*** and an ***end point***.



Properties of a vector:

In elementary mathematics, a ***vector*** is a geometric object ***defined by its properties***:

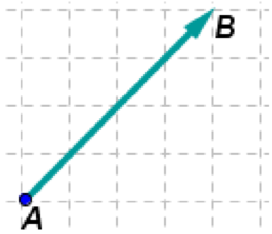
a) ***Direction:*** the line that ***holds*** $\vec{AB}$ or every line ***parallel*** to (AB); ***e.g.*** vertical, along line (d)...

b) ***Sense:*** orientation from an initial point to a final point; ***e.g.*** left to right or from A to B…

c) ***Magnitude:*** modulus or norm $\|\vec{AB}\|$ of the given vector is the distance between A & B.

Representation of a vector:

A vector is frequently represented by a ***segment with a definite direction***, or graphically as an ***arrow***, connecting an ***initial point*** A with a ***terminal point*** B, and denoted by $\vec{AB}$.



Special forms of Vectors ?

a) *Zero Vector:*

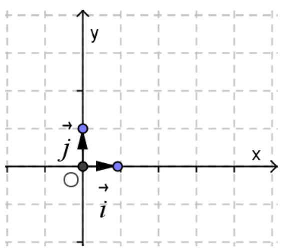
Def: Is a vector whose origin and extremity are confounded and it is also known as a ***null vector***.

Notation: A zero vector is denotedd by: $\vec{0}$ or $\overrightarrow{AA} = \overrightarrow{RR} = \overrightarrow{NN} = \vec{O}$

Properties: A zero vector has a zero modullus $\|\vec{0}\|$ and no definite direction.

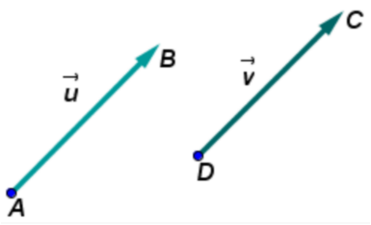
b) *Unit Vector:* is any vector whose magnitude is equal to the chosen unit (scale).

Eg: In the figure to the right $\vec{i}$ & $\vec{j}$ represent the unit vectors of $x-axis$ & $y-axis$ respectively, where $\|\vec{i}\| = \|\vec{j}\| = 1unit$



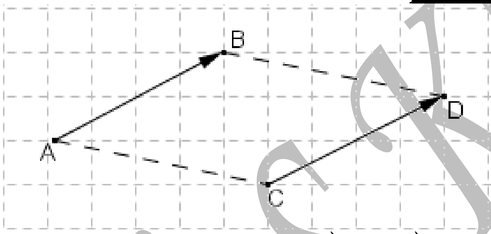
➪ Relating vectors:

☆ Equal vectors:

Analytic approach	Geometric approach	Conclusion
Any two vectors $\vec{u}$ *and* $\vec{v}$ are ***equal*** if they have: - Same direction. - Same sense. - Same magnitude.		***Therefore***, vectors $\vec{u}$ *and* $\vec{v}$ are ***equal***. And we write: $\vec{u}=\vec{v}$ OR $\vec{AB}=\vec{DC}$

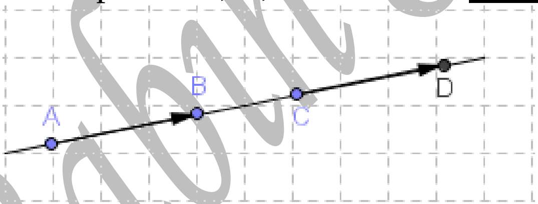
✓ **Significance of equal vectors:**

a. If $\vec{AB}=\vec{CD}$, then C is the fourth vertex of the ***parallelogram*** $ABDC$.

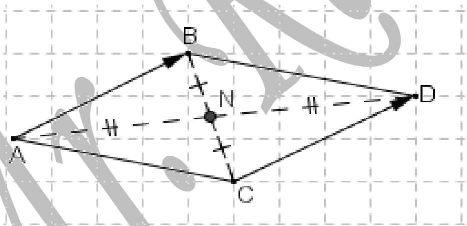


Conversely: If $ABDC$ is a ***parm*** then, $\vec{AB}=\vec{CD}$.

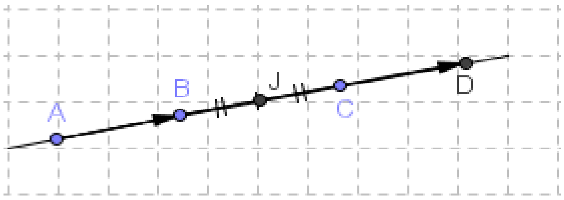
b. If $\vec{AB}=\vec{CD}$, then the points *A, B, C* and *D* are ***collinear***.



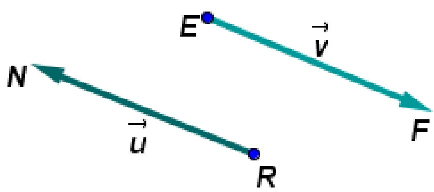
c. If $\vec{AB}=\vec{CD}$, then the segments $[AD]$ and $[BC]$ have the ***same midpoint***.



Or



☆ Opposite Vectors:

Analytic approach	Geometric approach	Conclusion
Any two vectors $\vec{u}$ *and* $\vec{v}$ are opposite if they have: - Same direction. - Same magnitude. - But opposite senses.		***Therefore***, $\vec{u}$ *and* $\vec{v}$ are ***opposite***. And we write: $\vec{u}=-\vec{v}$ OR $\vec{RN}=-\vec{EF}$ OR $\vec{RN}+\vec{EF}=\vec{0}$

Significanceof opposite vectors is the same as equal vectors

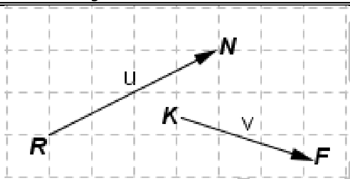
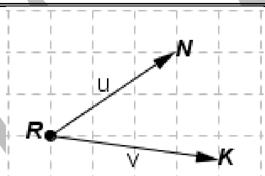
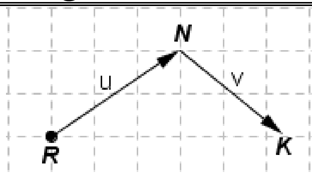
☆ *Collinear vectors* :

Vectors are ***collinear*** if and only if they have ***same direction***: *means* $\begin{cases} \text{held by same st.line} \\ \text{held by parallel st.lines.} \end{cases}$

In other words, two non-zero vectors $\vec{u}\ \&\ \vec{v}$ are collinear if they satisfy the vector relation:

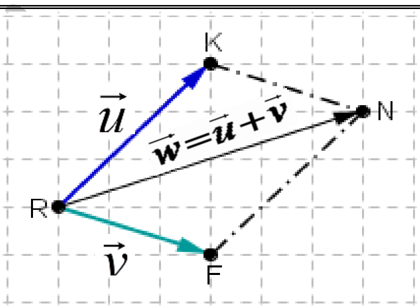
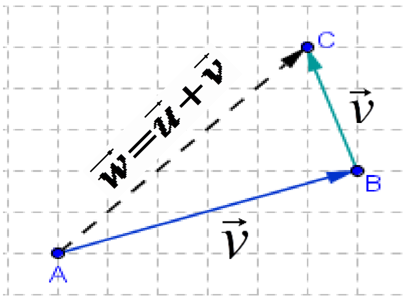
Relation	*Meaning*			
$\vec{u} = k\,\vec{v}$	*If* $k = 1$ *then*	*If* $k = -1$ *then*	*If* $k > 1$ *then*	*If* $k < 1$ *then*
	$\vec{u}\ \&\ \vec{v}$ are equal vectors	$\vec{u}\ \&\ \vec{v}$ are opposite vectors	$\vec{u}\ \&\ \vec{v}$ have same sense	$\vec{u}\ \&\ \vec{v}$ have opposite senses
$\lVert\vec{u}\rVert = \lvert k\rvert \lVert\vec{v}\rVert$				

➯ *Types of vectors*:

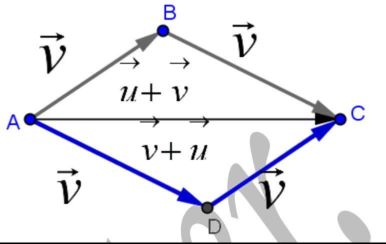
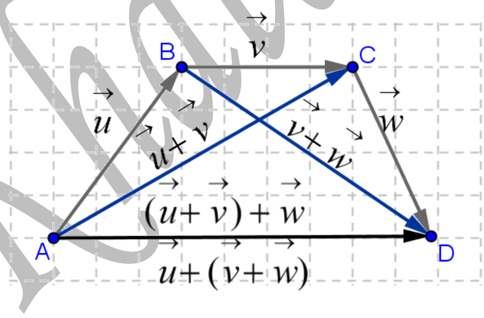
	Free vectors	*Vectors of the same origin*	*Consecutive Vectors*
Definition	Are vector having neither a common origin nor common extremity.	Are vector having a common origin only.	Are vectors having the extremity of the first as the origin of the second.
Geometric approach			

➯ *Sum of two vectors*:

There are two main methods to add two or more vectors having the same ***coefficients***:

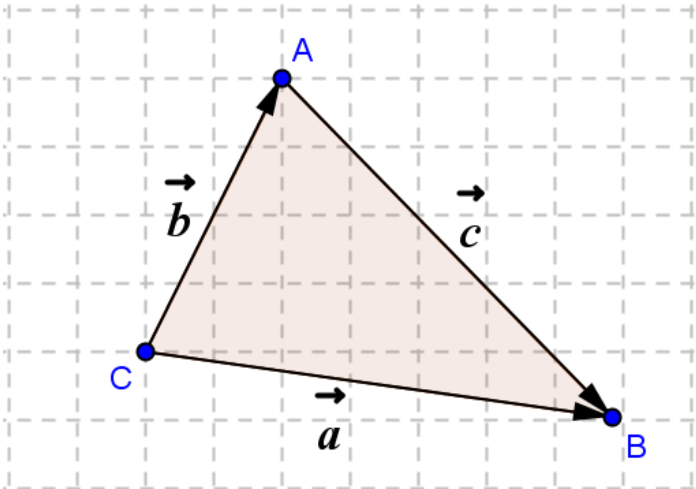
	1^{st} – *Method*	2^{nd} – *Method*
	Parallelogram Rule	***Chasles' Rule***
Used	*If vectors have the same origin*	*If vectors are consecutive*
Method	*Complete the parallelogram*	*Join the first origin to the last extremity.*
Graphical representation		
Analytical approach	The sum of two vectors with same origin; is a vector with same origin and its extremity is the fourth vertex of the parm. $\vec{RF} + \vec{RK} = \vec{R\underbrace{N}_{4thVertex}}$.	The sum of two consecutive vectors; is a vector with orgin of 1^{st} and extremity of the last. $\vec{AB} + \vec{BC} = \vec{AC}$.

Properties of vector addition:

No.	Analytic approach	Geometric approach
1.	Commutativity: $Since, \left.\begin{array}{l}\vec{u}+\vec{v}=\vec{AC}\\ \vec{v}+\vec{u}=\vec{AC}\end{array}\right\} so, \vec{u}+\vec{v}=\vec{v}+\vec{u}$	
2.	Associativity: $Since, \left.\begin{array}{l}\vec{u}+\vec{v}=\vec{AC}\\ \vec{AC}+\vec{w}=\vec{AD}\end{array}\right\} then, (\vec{u}+\vec{v})+\vec{w}=\vec{AD}$ $Since, \left.\begin{array}{l}\vec{v}+\vec{w}=\vec{BD}\\ \vec{u}+\vec{BD}=\vec{AD}\end{array}\right\} then, (\vec{v}+\vec{w})+\vec{u}=\vec{AD}$ Thus, $(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$	

Triangular inequality:

In any triangle ABC we have:



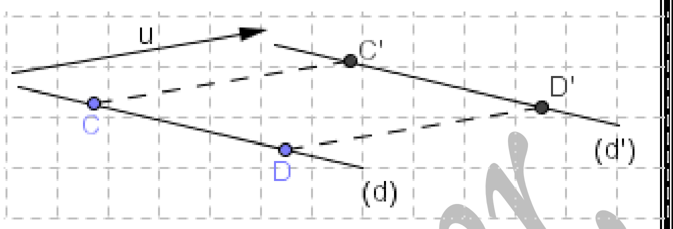
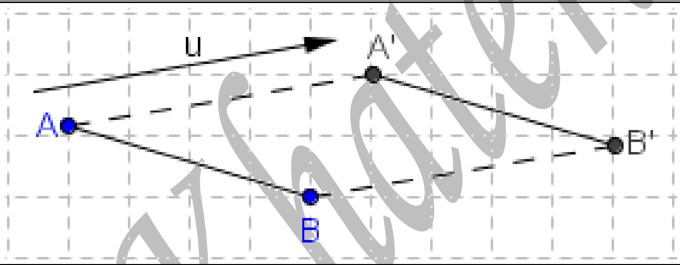
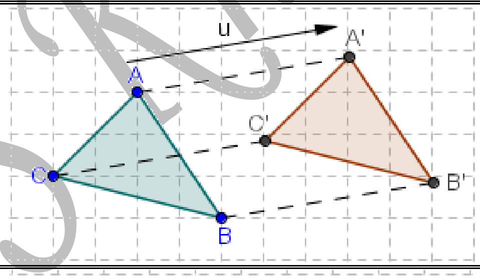
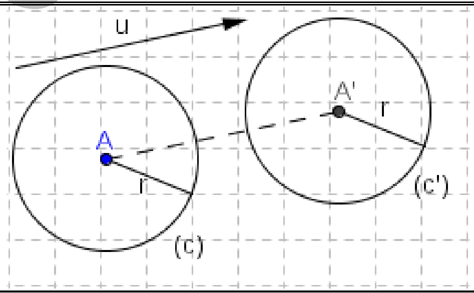
$\|\vec{a}\| \le \|\vec{b}\| + \|\vec{c}\|$

But, $\vec{b}+\vec{c}=\vec{a}$

So, $\|\vec{b}+\vec{c}\| = \|\vec{a}\|$

$Thus, \|\vec{b}+\vec{c}\| \le \|\vec{b}\| + \|\vec{c}\|$

How to find the Image or translation of some geometric figures

Figures	Procedure	Graphical representation
Lines	To translate a line, translate any two points on this line.	
Segments	To translate a segment, translate its extremities.	
Triangles	To translate a triangle, translate its vertices	
Circles	To translate a circle, translate its center and keep the same radius	

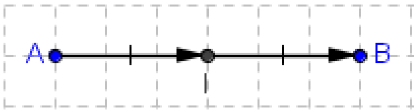
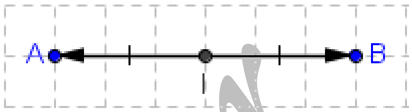
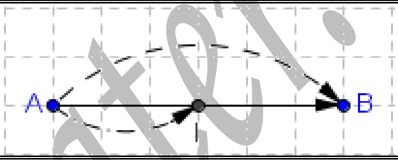
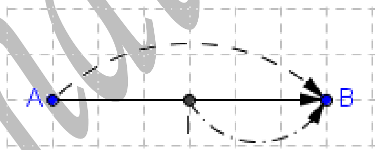
➪ ***Properties of translation:***

Translation preserves:

- Parallelism and orthogonality
- Areas and congruency
- Length and midpoints
- Angles
- Collinearity

Midpoints and Vectors :

If I is the **midpoint** of $[AB]$ then;

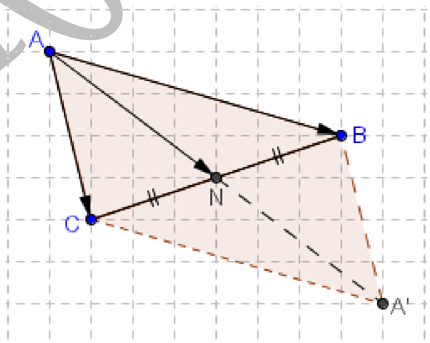
	Analytic approach	Graphical representation
1-	$\vec{AI}=\vec{IB}$.	
2-	$\vec{IA}+\vec{IB}=\vec{0}$.	
3-	$\vec{AB}=2\vec{AI}$.	
4-	$\vec{AB}=2\vec{IB}$.	

☆ *Conversely:*

$$\text{If } \begin{cases} \vec{AI}=\vec{IB} \\ \vec{IA}+\vec{IB}=\vec{0} \\ \vec{AB}=2\vec{AI} \\ \vec{AB}=2\vec{IB} \end{cases} \text{ then, } \boldsymbol{I} \text{ is the } \textbf{midpoint} \text{ of } [\boldsymbol{AB}]$$

Medians and Vectors :

If $[AN]$ is a median relative to $[BC]$ then; $\vec{AB}+\vec{AC}=2\vec{AN}$.



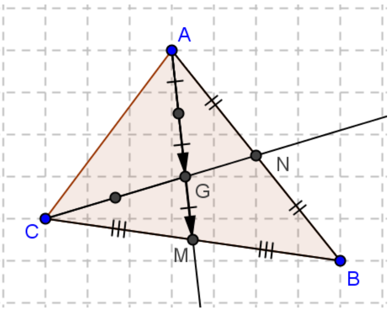
☆ *Conversely:* If $\vec{AB}+\vec{AC}=2\vec{AN}$ then, $[AN]$ is the median relative $[BC]$.

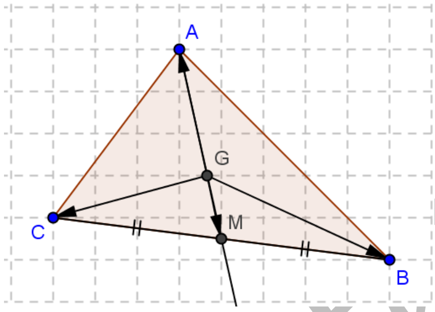
☆ *Genrerally:* If A is any point in the plane and M is the midpoint of $[BC]$ then we write:

$$\boxed{\vec{AB}+\vec{AC}=2\vec{AM}.}$$

Centroid and Vectors:

If **G** is the center of gravity (***Centroid***) of triangle ***ABC*** then:





$\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$

$\overrightarrow{GM} = \frac{1}{3}\overrightarrow{AM}$

$\overrightarrow{AG} = 2\overrightarrow{GM}$

$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \overrightarrow{0}$

Proof:

$\overrightarrow{GB} + \overrightarrow{GC} = 2\overrightarrow{GM}$ (Midpoint of a segment)

$But, 2\overrightarrow{GM} = -\overrightarrow{GA}$

$Thus, \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \overrightarrow{0}$

☆ *Conversely*: ***If*** $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \overrightarrow{0}$ then; ***G*** is the ***center of gravity*** of triangle ***ABC***.

☆ *Genrerally: If* M is any point of the plane where G is the centroid of ΔABC then we can write:

$\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = 3\overrightarrow{MG}$

Proof: Since G is the centroid of ΔABC then

$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \overrightarrow{0}$ (Introduce point M .)

$\overrightarrow{GM} + \overrightarrow{MA} + \overrightarrow{GM} + \overrightarrow{MB} + \overrightarrow{GM} + \overrightarrow{MC} = \overrightarrow{0}$

So, $3\overrightarrow{GM} + \overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = \overrightarrow{0}$

$\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = -3\overrightarrow{GM}$

Therefore;

$$\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = 3\overrightarrow{MG}$$

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