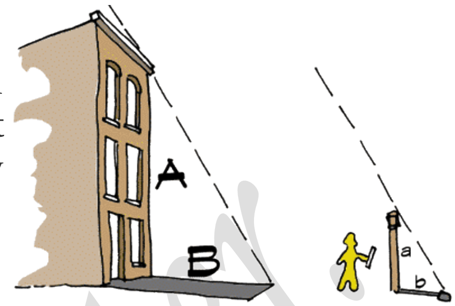
Introduction:

The building of the Egyptian pyramids may seem to have little in common with devising modern radar, air-planes and H-bombs. But certain principles of mathematics enter into all such activities. Many are used in the field of mathematics called ***Trigonometry***.

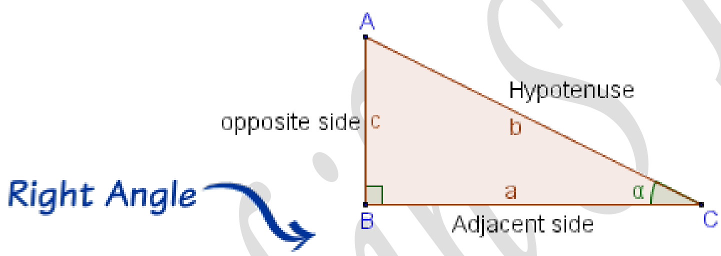


A. Goals:

1- *Find measure of an angle using radian.*
2- *Convert between units of angles.*
3- *Determine the length of an arc.*
4- *Define a trigonometric circle.*
5- *Construct oriented arcs.*
6- *Define trigonometric lines and their properties.*

B. A profound review:

1. Labeling the sides of a right-angled triangle:



2. Defining the three main trigonometric ratios: $\sin\alpha; \cos\alpha; and \tan\alpha$.

Consider the acute angle α enclosed between the two fixed rays $[ox)$ *and* $[oy)$. Let A be a variable point on ray $[oy)$ and B is its orthogonal projection on $[ox)$.

a. Is α constant? Justify. ……………………………………………………………..
b. Indicate how does (AB) vary with respect to $[Ox)$? ………………………………………
c. Prove that: $\frac{OB}{OA} = \frac{OB_1}{OA_1}$ ……………………………………………………………

……………………………………………………..

d. Prove that: $\frac{OB}{OA} = \frac{OB_2}{OA_2}$ …………………………

……………………………………………………..

e. Complete: $\frac{OB}{OA} = ... = ... = \frac{OB_3}{OA_3} = cst$

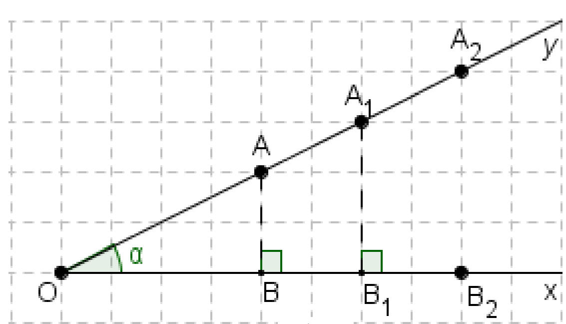


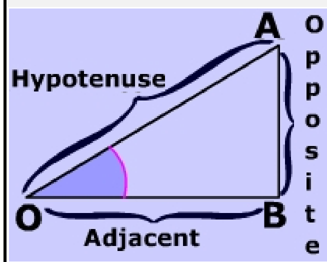
Fig-1.

f. Does these ratios depend on the:
   i. Position of A on ray $[oy)$? ……………………………………………
   ii. Value angle α ? ………………………………………………………..

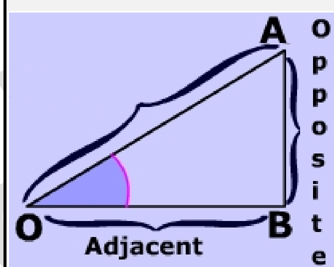
Attention!!! In this year we are not only concerned with acute angles of a right triangle.

3. ***Trigonometric ratios:***

Cosine and Sine ratios:

Cosine ratio		Reference triangle	Sine ratio	
To remember rule Cah	$\cos A\hat{O}B = \frac{Adjacent}{\text{Hypotenuse}} = \frac{OB}{OA}$		$\sin A\hat{O}B = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{OA}$	**To remember rule** soh
	IN SHORT $\cos\alpha = ...$		**IN SHORT** $\sin\alpha = ...$	

Tangent and co-tangent Functions:

Tangent ratio	Reference triangle	Cotangent ratio
$\tan A\hat{O}B = \frac{\text{Opposite}}{Adjacent} = \frac{AB}{OB}$		$\cot A\hat{O}B = \frac{Adjacent}{\text{Opposite}} = \frac{OB}{AB}$
IN SHORT $\tan\alpha = \frac{Opp}{adj}$ OR $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$		**IN SHORT** $\cot\alpha = \frac{adj}{Opp}$ OR $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ $\cot\alpha \cdot \tan\alpha = 1$
Toa OR tsc	**To remember rule**	Ccs

C. ***Fundamental trigonometric identities relating:***

1) Sine and Cosine: The Pythagorean identity: $\sin^2\alpha + \cos^2\alpha = 1$

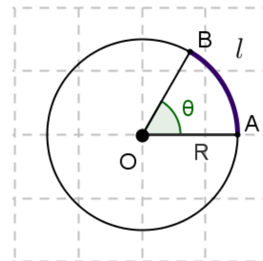
2) Cosine and tangent: $\tan^2\alpha = \frac{1}{\cos^2\alpha} - 1$ $\cos^2\alpha = \frac{1}{\tan^2\alpha + 1}$

3) sine and cotangent: $\cot^2\alpha = \frac{1}{\sin^2\alpha} - 1$ $\sin^2\alpha = \frac{1}{\cot^2\alpha + 1}$

D. Length of an arc:

Consider the following table:

Length of an arc		Measure of central angle
Greatest arc $P_{circle} = 2\pi R$	Corresponds to $\longrightarrow$	***Greatest central angle*** 360^o
vA length l		θ



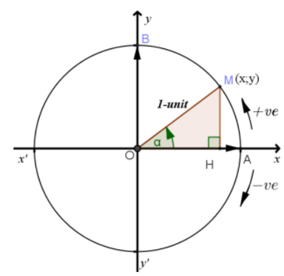
From the above table we notice that:
The ***length*** of an arc intercepted by a central angle expressed in:

Degrees	Radians
$l = \dfrac{2\pi R\theta}{360^\circ}$	$l = R\beta$
Where, θ is expressed in degrees	Where β is expressed in radians

E. *Trigonometric circle:*

Def: is a circle with one unit radius and a definite positive direction called the direct sense, which is the ***anti-clock*** wise sense.

In the trigonometric circle $\left(O, \overrightarrow{OA}, \overrightarrow{OB}\right)$, A is the origin of arcs.



Remarkable angles:

Degrees versus radian										
α : in degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°
β : in radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$

From the table

If α and β

Quadrant	1^{st} - quadrant				
Complements					
Angle (α) / Ratios	$0\,rad$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
$\cot\alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

We notice that

Complementary

$\alpha + \beta = \frac{\pi}{2}$

$\cot\alpha \times \cot\beta = 1$

$\tan\alpha \times \tan\beta = 1$

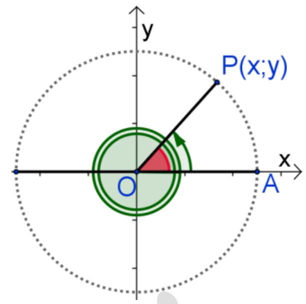
$\tan\alpha = \cot\beta$

$\cos\alpha = \sin\beta$

F. Principal determination:

Principal angle is the measure of the oriented arc AP that belongs to the interval:

$]-\pi,\pi]$	$]-180^{\circ},180^{\circ}]$
If angle is in radian	If angle is in degrees



G. Trigonometric lines:

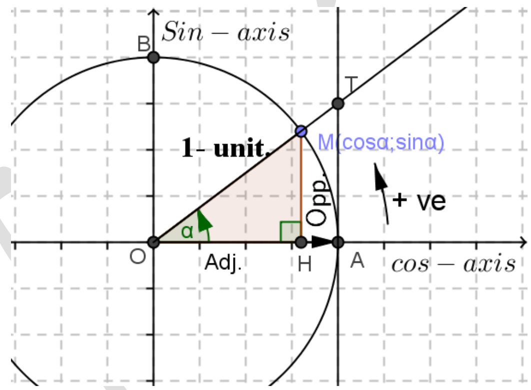
Consider the portion of the trigonometric circle $\left(O,\overrightarrow{OA},\overrightarrow{OB}\right)$

Form the adjacent figure notice that:

1- Horizontal axis represents cosine $-$ *axis*

2- Vertical axis represents sine $-$ axis

3- $\overline{OH}=\cos\alpha$ $\quad\overline{HM}=\sin\alpha$ $\quad\overline{AT}=\tan\alpha$



H. Bounding (Framing) trigonometric ratios:

<table>
<tr><th colspan="4">Bounding trigonometric lines</th></tr>
<tr><td>$|\cos\alpha|\le 1$</td><td rowspan="2">means</td><td>$-1\le\cos\alpha\le 1$</td><td rowspan="2">$\forall\ \alpha\in\mathbb{R}$</td></tr>
<tr><td>$|\sin\alpha|\le 1$</td><td>$-1\le\sin\alpha\le 1$</td></tr>
<tr><td colspan="4">$\left.\begin{array}{l}-\infty<\tan\alpha<+\infty\\-\infty<\cot\alpha<+\infty\end{array}\right\}\forall\ \alpha\in\mathbb{R}$</td></tr>
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