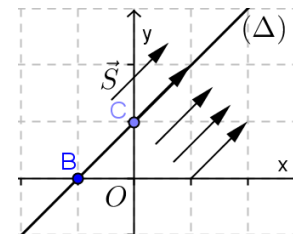


Name:

"Lines in a Plane"

S.S-5

- A direction vector \vec{S} of any straight line (Δ) is a **non-zero** vector having the same direction (*parallel* or *confounded*) as (Δ).
- A straight line (Δ) admits an infinite number of **non-zero** direction vectors collinear with \vec{S} .
- To determine the intersection of a straight line with:
 - 1) *x* - axis : (*x*-intercept) Substitute $y = 0$.
 - 2) *y* - axis : (*y*-intercept) Substitute $x = 0$.
- To determine an equation of a straight line it is **sufficient** to have **one** of the following **combinations**:



Given information	Formula used and steps to follow	
Any two points $A(x_A; y_A)$ and $B(x_B; y_B)$	✓ Take any point $M(x, y)$ on the given straight line. Now, points A, B & M are collinear, Then $a_{(AM)} = a_{(AB)}$ Thus, $\frac{y - y_A}{x - x_A} = \frac{y_B - y_A}{x_B - x_A}$ $\frac{y - y_A}{x - x_A} = m$ OR $y - y_A = m(x - x_A)$	
A point $A(x_A; y_A)$ and a slant (leading coefficient or director coefficient)	1. Take any point $M(x, y)$ on the given straight line. 2. Form the vector: $\vec{AM}(x - x_A, y - y_A)$ 3. Since \vec{S} & \vec{AM} are collinear then: <u>In cartesian:</u> $\frac{X_{\vec{AM}}}{X_{\vec{S}}} = \frac{Y_{\vec{AM}}}{Y_{\vec{S}}}$ <u>In parametric:</u> $\vec{AM} = t\vec{S}$	
Any point $A(x_A; y_A)$ and a director vector $\vec{S}(x', y')$	1. Take any point $M(x, y)$ on the given straight line. 2. Form the vector: $\vec{AM}(x - x_A, y - y_A)$ 3. Since \vec{N} & \vec{AM} are perpendicular (orthogonal) then: $\vec{N} \cdot \vec{AM} = 0$ OR $a(x - x_A) + b(y - y_A) = 0$	
Any point $A(x_A; y_A)$ and a normal vector $\vec{N}(a; b)$	1. Take any point $M(x, y)$ on the given straight line. 2. Form the vector: $\vec{AM}(x - x_A, y - y_A)$ 3. Since \vec{N} & \vec{AM} are perpendicular (orthogonal) then: $\vec{N} \cdot \vec{AM} = 0$ OR $a(x - x_A) + b(y - y_A) = 0$	

Different types of equation of a straight line and their properties

Form	Equation	Director vector	Normal vector	Slant	Find a point on the st. line
<i>Cartesian</i>	$ux + vy + w = 0$	$\vec{S}(-v; u)$	$\vec{N}(u; v)$	$\frac{-u}{v}$	Take any value for x then find y
<i>Reduced</i>	$y = mx + b$	$\vec{S}(1; m)$	$\vec{N}(m; -1)$	m	Take any value for x then find y
<i>Parametric</i>	$\begin{cases} x = at + x_A \\ y = bt + y_A \end{cases}$	$\vec{S}(a; b)$	$\vec{N}(-b; a)$	$\frac{b}{a}$	Take any value for the parameter to find x and y
<i>Special</i>	$x = cst = r.$	$\vec{S}(0; 1) = \vec{j}$	$\vec{N}(1; 0) = \vec{i}$	No slope	Any point of the form $(r; y)$
	$y = cst = n.$	$\vec{S}(1; 0) = \vec{i}$	$\vec{N}(0; 1) = \vec{j}$	Zero slope	Any point of the form $(x; n)$

❖ How to interchange between forms of a straight line?

To modify an equation of a straight line from:

To change from	Cartesian to parametric	parametric to Cartesian
<i>Procedure</i>	<p>1st - Step: Replace one of the variables by any parameter "t".</p> <p>2nd - Step: Find the other variable in terms of this parameter.</p>	<p>Just eliminate the parameter "t" by any way possible:</p> <ul style="list-style-type: none"> - Elimination. - Substitution. - Comparison.
<i>Example</i>	<p>(d): $2x - y + 5 = 0$</p> <p>1st - step: Let $x = t$.</p> <p>2nd - step: Replace $x = t$ in equation of (d)</p> $2t - y + 5 = 0$ <p>Hence, $y = 2t + 5$.</p> <p>Thus, the parametric form of straight line is,</p> $(d): \begin{cases} x = t \\ y = 2t + 5 \end{cases}$	<p>By elimination:</p> $(d): \begin{cases} x = t - 4 \\ y = t + 5 \end{cases}$ <p>To get: (d): $x - y = -9$</p>

Relative positions of two straight lines in different forms

<i>Form</i>	<i>Equations</i>	<i>If</i>	<i>Then, the straight lines</i>
<i>Cartesian</i>	$(d_1): ux + vy + w = 0.$ $(d_2): u'x + v'y + w' = 0.$	$\frac{u}{u'} \neq \frac{v}{v'}$	$(d_1) \& (d_2)$ are <i>intersecting</i> .
		$\frac{u}{u'} = \frac{v}{v'} \neq \frac{w}{w'} \quad \text{OR} \quad \vec{S}_1 = k\vec{S}_2$	$(d_1) \& (d_2)$ are <i>parallel</i> .
		$\frac{u}{u'} = \frac{v}{v'} = \frac{w}{w'}$	$(d_1) \& (d_2)$ are <i>confounded</i> .
<i>Reduced</i>	$(R_1): y = mx + b.$ $(R_2): y = m'x + b'.$	$m \neq m'$	$(R_1) \& (R_2)$ are <i>intersecting</i> .
		$m = m' \quad \text{and} \quad b \neq b'.$	$(R_1) \& (R_2)$ are <i>parallel</i> .
		$m = m' \quad \text{and} \quad b = b'.$	$(R_1) \& (R_2)$ are <i>confounded</i> .
<i>Parametric</i>	$(P_1): \begin{cases} x = at + x_A \\ y = bt + y_A \end{cases}$ $(P_2): \begin{cases} x = a't + x_B \\ y = b't + y_B \end{cases}$	$\frac{a}{a'} \neq \frac{b}{b'}$	$(P_1) \& (P_2)$ are <i>intersecting</i> .
		$\frac{a}{a'} = \frac{b}{b'} \neq \frac{x_A - x_B}{y_A - y_B}$	$(P_1) \& (P_2)$ are <i>parallel</i> .
		$\frac{a}{a'} = \frac{b}{b'} = \frac{x_A - x_B}{y_A - y_B}$	$(P_1) \& (P_2)$ are <i>confounded</i> .