Al Mahdi High Schools (Al-Hadath)	Mathematics	10th-Grade
Name:	"Relation versus Function"	S.S-7

✓ **Introduction**:

The relation-function concept is one of the most important ideas in mathematics, and consequently in our everyday life.

- A football team should include ***at least*** one defender.
- In a football match each team has ***at most*** one goalkeeper position.
- For each car ***corresponds,*** four wheels.
- To each person ***corresponds,*** a specific age.

Give four mathematical examples that includes the terms at ***least*** or at ***most*** (2 for each)

……………………………………………. ……………………………………………….

……………………………………………. ……………………………………………….

To each month of the year ***corresponds,*** one and only one name.

> The importance of ***correspondence*** lies in prediction of other terms of the relation.

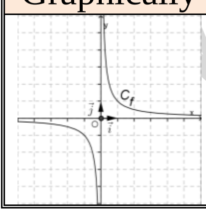
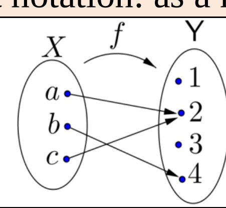
Eg $_1$: An engineer tries to find the correspondence (relation) between the car's performance and the number of kilometers it covers.

➢ Example: A tree grows 20 cm per year.

i. ***Find*** the height of the tree: $\begin{cases} a)\ 3\,years\,later :.. \\ b)\ 10\ years\,later :...................................... \end{cases}$

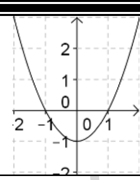
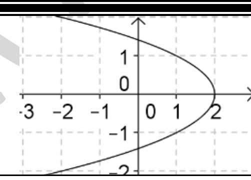
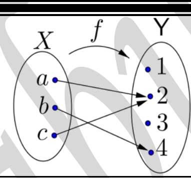
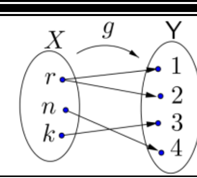
ii. ***Relate*** the height of the tree to its age. ………………………………………………..

Relations and Functions can be represented in four interrelated ways:

1	2	3	4
Graphically	As ordered pairs	In explicit form: as an equation	In a set notation: as a mapping
	$(2,1),(-1,0)$	$y^2 = x^2 - 3$ $y = x^2 - 3$	

Relation versus functions

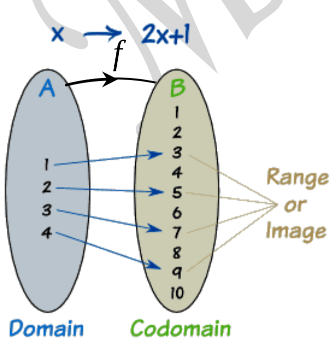
Study carefully the following table to figure out the definition of a function and that of a relation:

	Examples of functions	Examples of relations
Ordered pair	$N=\{(-1;1),(1;1),(2;4),(3;9)\}$	$R=\{(-3;2),(0;1),(2;5),(0;4)\}$
	List the set of values of the 1st components (abscissas: x)	
	Does there exist an abscissa, x, for which corresponds two ordinates y?	
Explicit form As an equation	$y=x+1$	$y^2=x$
	Find the values of y, for $x=1,4\&9$	
	What do you notice?	
Graphically		
	Does there exist an x for which corresponds two values of y?	
Mapping		
	Does there exist an x for which corresponds two values of y?	

Complete the following definitions:

- A relation is a rule that assigns ………………………………………………………………
- A function is a rule that assigns ………………………………………………………………

Terminologies:



1- f : represents the ***name*** of the ***function*** (rule).
2- $f(x)$: represents the ***image*** of x given by the function f.
3- The variable x is called the ***independent*** variable.
4- The set of values x, for which there ***correspond*** a value of y is called the domain of the given function f.
5- The variable $y=f(x)$ is called the ***dependent*** variable.
6- The set of values $y=f(x)$ is called the range of the given function f.

❖ Mathematical Relation:

Def₁	A relation is a ***rule*** (process or method) that produces a ***correspondence*** between an ***<u>initial</u> <u>set</u> of elements*** called the ***<u>domain</u>*** and a ***<u>final set</u> of elements*** called the ***<u>range</u>***, such that for each element in the domain corresponds at ***<u>least one</u>*** element in the range.

Def₂	A relation is any set of ***ordered pairs***. The set of all 1st- components of the ordered pairs is called the ***domain***, and the set of all 2nd- components is called the ***range*** of the relation.

Ex₁: Consider the relation between the width (x) and the length (y) of a rectangle, defined by the expression, $y = 2x - 1$.

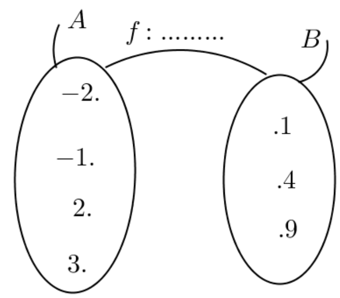
a) Find for each value of $x, such\, that\, x = \{1, 2, 5, 7\}$ the corresponding value of y.

Pre-image	x		2		
Image	y		3		

b) What does the set $D = \{1, 2, 5, 7\}$ represent? …………………………………………………...

c) Find set of range, R, of the given relation. ……………………………………………………..

Ex₂: Consider the following Venn diagram:



a) Write the relation that exists between the given sets.

b) Write in extension the set of:

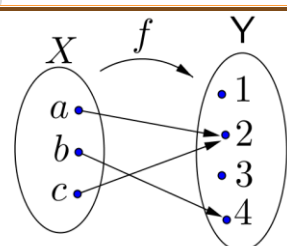
i. Domain:…………………………………………

ii. Range: …………………………………………

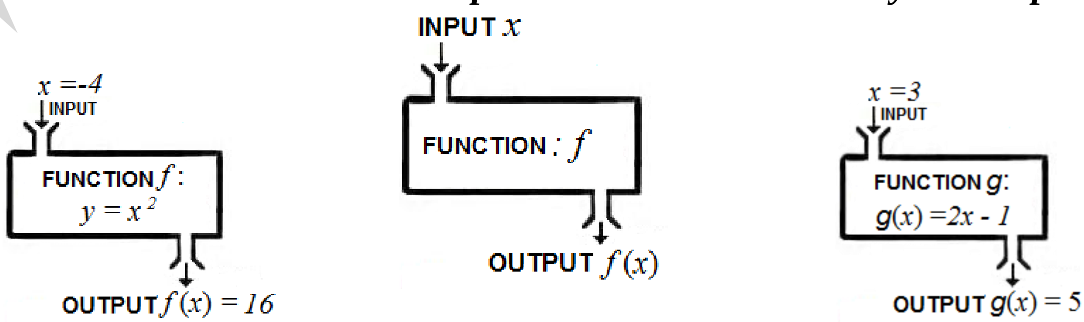
c) What do the values $-2 \& 4$ represent in the given relation?

❖ **Function:**

Def	A function is a relation that assigns ***for each element*** x in the set of domain ***one and only one element*** y in the set of range.



Function machine: For each ***input*** value there exists ***exactly one output*** value only.



Ex$_3$: Consider the relation $f: x^2 + y^2 = 25$, assuming that x is the independent variable.

a. Find for the following values of $x = \{-1, 0, 2, 3\}$ the corresponding values of $y = f(x)$.

………………………………… ………………………………………..

………………………………… ………………………………………..

b. Is the above relation a function? Justify.

………………………………………………………………………………………..

………………………………………………………………………………………..

How to determine that a given curve is a function or not?

✓ Vertical line test:

We can use the graph of a given relation to determine if this relation defines a function or not.

To check if the graph of a given relation represents a function we perform a test called the ***vertical line test***.

If any vertical line cuts the graph of a given relation in at most one point, then the relation is a function.

Ex$_3$: Use the vertical line test to justify which relation represents a function.