## The domain of definition of a function can be determined:

### 1) Algebraically:

Let $g$ be the function defined by $g(x)=\frac{p(x)}{q(x)}=\frac{(x+1)(x+3)}{x^2-3x+2}$.

***a-*** Complete the following table:

| $x$ | $p(x)$ | $q(x)$ | $g(x)$ |
|---|---|---|---|
| 0 | | | |
| 2 | | | |
| -2 | | | |
| 1 | | | |

***b-*** For which values of $x$, does the denominator of $g(x)$ vanish?……………………………

***c-*** For which values of $x$, is $g(x)$ undefined? (Can't be calculated) …………………………..

***d-*** Deduce the domain of $g$. ………………………………………………………………………

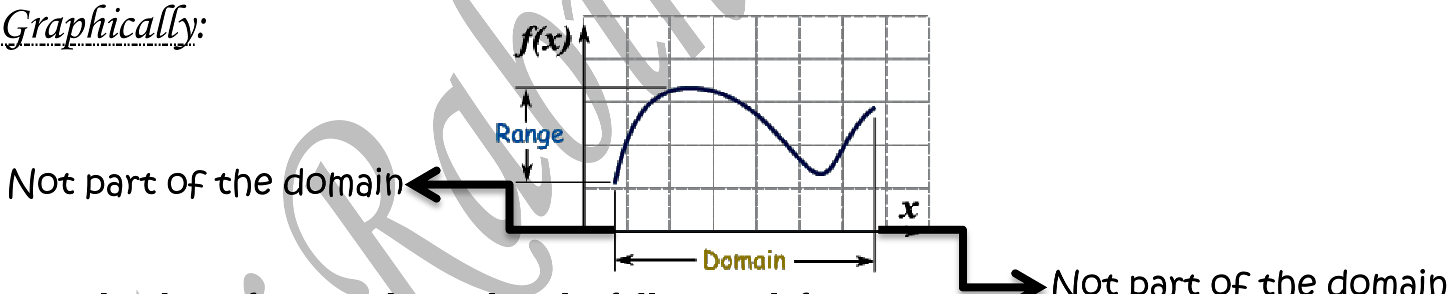
***e-*** Is $g$ defined over the interval $I=[3;+\infty[$ ? Justify. ………………………………………………..

……………………………………………………………………………………………………

***f-*** Does $x=2^{1000}$, admit an image by $g$ ? Explain. ………………………………………………

- Define the ***domain***: ………………………………………………………………………………
- ***Range*** is the inverse of domain.
- Define the ***Range***: ………………………………………………………………………………..

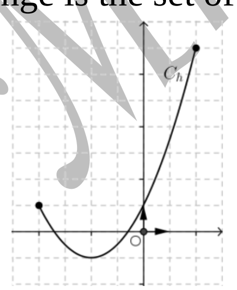
### 2) Graphically:

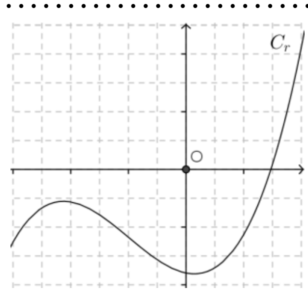


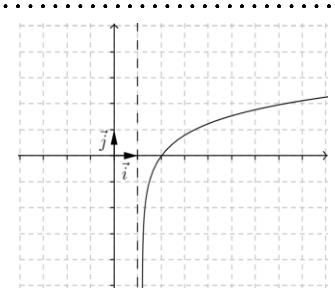

Observe the above figure and complete the following definitions:

1- Domain is the set of values of ……………………………………………………………………..

2- Range is the set of values of ………………………………………………………………………




$D_h=$……………………… $D_r=$……………………… $D_f=$………………………

$R_h=$……………………… $R_r=$……………………… $R_f=$………………………

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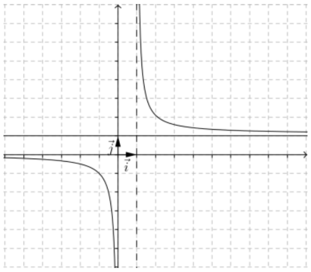
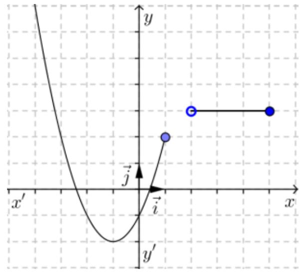
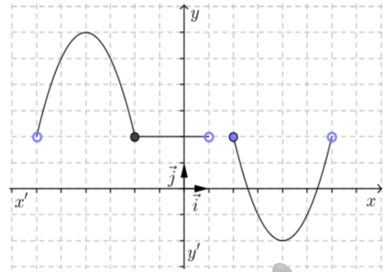
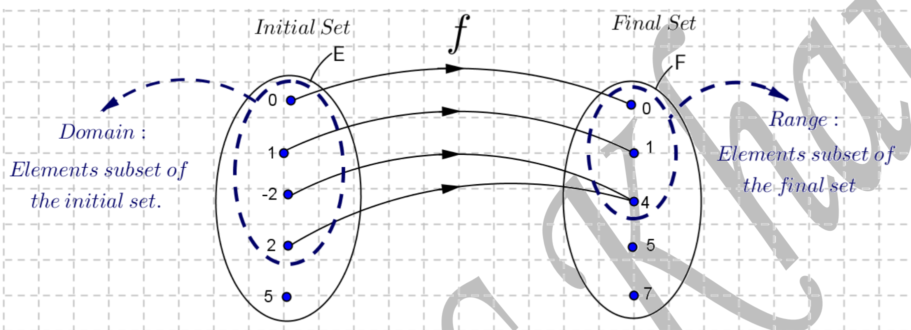




$D_g =$........................................ $D_s =$............................ $D_k =$............................

$R_g =$........................................ $R_s =$............................ $R_k =$............................

## 3) Using sets:

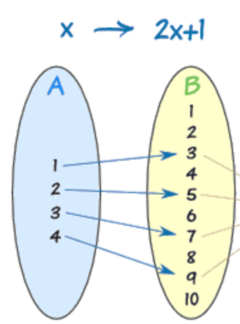



## Answer the following:

- *Write* in extension the:
  - ***Initial*** set $E$: $E = \{$..............................$\}$
  - ***Final*** set $F$ (Co-domain): ……………………………………………………………
- Each element $x$ of the set $E$, for which corresponds a value of $y$ is denoted by the ***pre-image*** (antecedent) or the ***independent*** variable.

  List any two pre-images:
- Each element $y$ of $F$, for which corresponds at least one value of $x$ is denoted by the ***image*** or the ***dependent*** variable.

  List any two images:




For the adjacent figure:

- Give the pre-images of 3: …… and of 5: ……..
- Indicate the images of 3: …… and of 4: ……..
- What can you say about element 10 of set B?
- Find the set of ***domain*** of $f$ : ……………………
- Find the set of ***range*** $f$ : ………………………..

***Def:*** Domain is the set of all values of the *independent* variable $x$ in the *initial* set $E$ which corresponds to it a real (defined) *image* $y$ in the *final* set $F$.

***Def:*** Range is the set of all values of the *dependent* variable $y$ in the *final* set $F$ which corresponds to a *pre-image* $x$ in the set *initial* $E$.

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## 4) Using ordered pairs:

Consider the set of ordered pairs: $P = \{(-3,2),(0,1),(2,5),(-1,4)\}$.

a) Write in extension the set:

i. $D_P$, the set of all values of $x$: ……………………………………..

ii. $R_P$, the set of all values of $y$: ……………………………………..

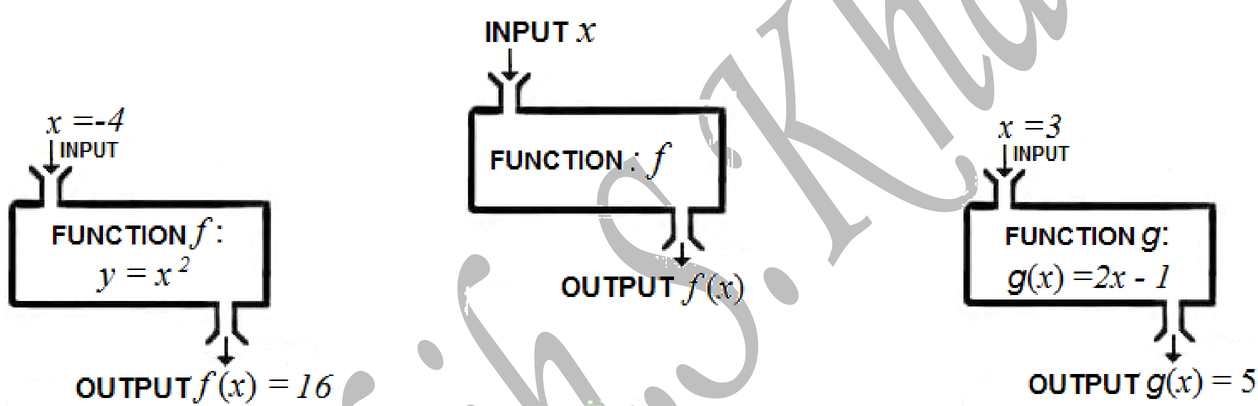
b) Define the:

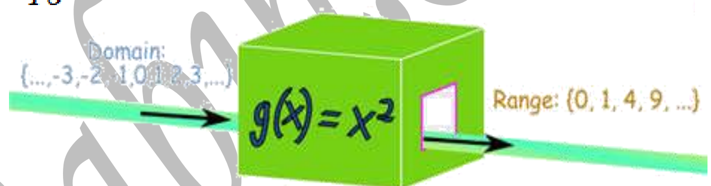
i. Domain: …………………………………………………………………

ii. Range: ……………………………………………………………………..

## 5) Analytically (by calculation):

**Reminder: Function machine:** For each ***input*** value there exists ***exactly one output*** value only.




Analytically domain is defined as the set of real values of *x* (inputs) for which corresponds a real value of *y* (output).

| ***a.*** Polynomial | Are functions of the form: $P(x) = ax^n + bx^{n-1} + cx^{n-2} + ... + f$ |
|---|---|
| ***Domain:*** | Is of the form: $D_f = \Re = ]-\infty; +\infty[$ |
| $1 - f(x) = 3x^2 - x + 5$ | |
| $2 - g(x) = (2x-1)^3 - 2.$ | |
| $3 - q(x) = x^2 - (x+2)^2 - 2.$ | |

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| **b.** Rational function: eg. $p(x)=\frac{f(x)}{g(x)}=\frac{ax+b}{Ax^n+Bx+C}$. | Domain of definition is the set of all real numbers except for the values of $x$ that vanishes $g(x)$. That is $g(x)\neq 0$. |
|---|---|
| $1-\ t(x)=\frac{2}{x-1}$. | |
| $2-\ r(x)=\frac{x+2}{x(x+3)}$ | |
| $3-\ s(x)=\frac{x^2-4}{x^2+3}$. | |
| $4-\ h(x)=\frac{x^2+1}{x^2-5x+6}$. | |

| c. Irrational function: eg. $A(x)=\sqrt{f(x)}=\sqrt{ax^n+bx+c}$. | The domain of an irrational function is the set of all real numbers, such that $ax^2+bx+c\geq 0$. |
|---|---|
| $1-t(x)=\sqrt{3x+12}$. | |
| $2-r(x)=\sqrt{4-2x}$ | |
| $3-\ s(x)=\frac{\sqrt{x^2-4}}{x^2+3}$. | |
| $4-\ h(x)=\frac{x^2+1}{\sqrt{x^2-5x+6}}$. | |
| $5-\ p(x)=\sqrt{-2x+4}-\sqrt{x-3}+\sqrt[3]{x^2-1}$ | |

**Ex**: Determine the domain of definition of each of the following:

1) $y=\frac{3x-5}{x^2-3x+2}$ 2) $y=\frac{x+12}{x^2-4x+4}$ 3) $y=\frac{2x-3}{3x^2+1}$

4) $y=\frac{1}{|x-2|}$ 5) $y=\frac{x}{|x|-3}$ 6) $y=\frac{7}{|x-1|+5}$

7) $y=\frac{x-1}{|x^2-2|-7}$ 8) $y=\frac{2x+1}{\sqrt{x-3}}$ 9) $y=\frac{-1}{\sqrt{5-|x|}}$

10) $y=\sqrt{\frac{x-2}{1-x}}$ 11) $y=\sqrt{|x|-3}$ 12) $y=\sqrt{x^2+4}$

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