Mathematics — 10^{th}-Grade

Name: . . . . — **"Parity of a Function"** — S.S-9

To study the parity of any function:
First check if its domain is centered at ***origin***, if

- Yes → ***Then, continue***
- Not → ***Then,*** function ***does not admit*** any ***parity***

Even – functions:

A function f is said to be ***even***, iff: $f(-x)=f(x)$ for every value of x in the ***domain*** of f .

✓ *Analytical Study*:

Consider the functions:

$f: x \longrightarrow x^2$ $\qquad$ $g: g(x)=|x|$

$h: x \longrightarrow 5$ $\qquad$ $r(x)=-3$

1- Determine with ***justification*** for each of the given functions its:

a. Domain of definition:

……………………………………………………………………………………

……………………………………………………………………………………

b. Center:

……………………………………………………………………………………

……………………………………………………………………………………

2- Can we check for the parity of the given functions? Justify.…………………………………

3- Work out the following:

a. $f(-x)=$............................ $\qquad$ ***c.*** $h(-x)=$............................

b. $g(-x)=$............................ $\qquad$ ***d.*** $r(-x)=$............................

4- Compare $f(-x), g(-x)$ & $h(-x)$ with $f(x), g(x)$ & $h(x)$ respectively.

……………………… $\qquad$ ……………………… $\qquad$ ………………………

5- State conditions for which function is even:

Cond-1: …………………………… $\qquad$ ***Cond-2***: ……………………………

6- Deduce the parity of each of the given functions:

……………………………………………………………………………………

7- Find :

a. $f(x)+g(x)$……………… $\qquad$ ***c.*** $f(x).g(x)$……………… $\qquad$ ***e.*** $\dfrac{f(x)}{g(x)}$…………………

b. $f(x)-g(x)$……………… $\qquad$ ***d.*** $f(x)+h(x)$…………… $\qquad$ ***f.*** $f(x)\cdot h(x)$ …………..

8- Which of the above represent even functions? Justify.

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………………………………………… $\qquad$ …………………………………………

From the above example we can say that:

Conclusions	*If f & g are even functions*	*then*	*form*	*Out come*	*Example*
	If f is even & k a non-zero constant,	Multiple of f			$-2x^2$, $3\lvert x\rvert$
	If f & g are even functions, where $g \neq 0$	The sum			…………..
		The product			……………
		The quotient	$\frac{f}{g}$		$\frac{3\lvert x\rvert}{x^2-3}$

Consider the following points:

1) Determine the coordinates of the given points.

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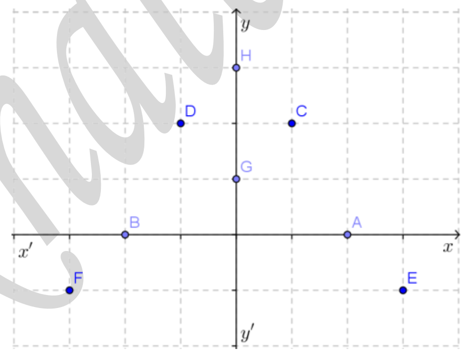
……….. ……….. ……….. ………..



2) Compare:

Points	Positions	Coordinates
A & B		
C & D		
E & F		
H & G		

3) Which of the above couples are symmetric with respect to $y-axis$?

……….. ……….. ……….. ………..

4) How can you tell that two points are symmetric with respect to $y-axis$?

……………………………………………………………………………………………

5) Complete the statement: Any two points with coordinates $(x; f(x))$ & $(-x; f(-x))$ are ..…………

✓ <u>*Graphical Study*</u>:

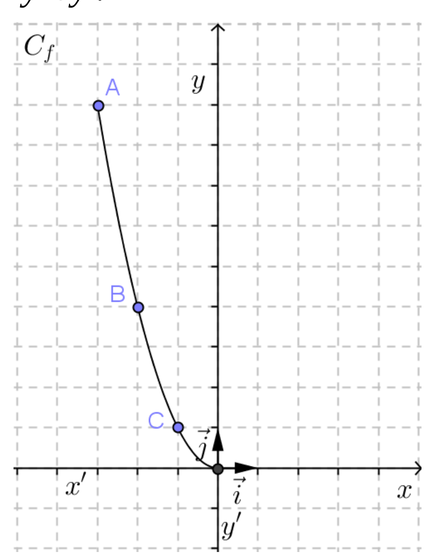
Consider a function f defined by a part of its graph (C_f) where the points $(A, B \& C) \in C_f$

1- Complete the ***symmetry*** of the points given points with respect to $y'oy$:

$$A(x=-3; f(x)= \quad)\xrightarrow{symm.w.r.t\ y'oy} A'(\quad;\quad)$$

$$B(\quad; f(x)=4)\xrightarrow{symm.w.r.t\ y'oy} B'(-x=2; f(-x)= \quad)$$

$$C(\quad;\quad)\xrightarrow{symm.w.r.t\ y'oy} C'(\quad;\quad)$$



2- Complete (C_f) by plotting the above points.

3- Suppose that f is defind by $f(x)=x^2$.

a. Is the function, f even? …………………

b. Is (C_f) symmetric with respect to $y-axis$? . . . .

4- What do you conclude? . . . . …………………..

Again, to study the parity of any function:

First check if its domain is centered at ***origin***, if
- Yes → ***Then, continue***
- Not → ***Then,*** function ***does not admit*** any ***parity***

<u>*Odd – functions*</u>

A function f is said to be ***odd iff*** : $f(-x)=-f(x)$ for every value of x in the domain of f .

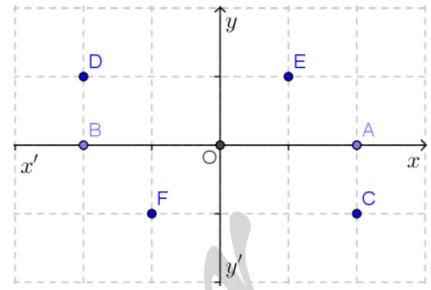
✓ <u>*Analytical Study*</u>:

Consider the functions $f:f(x)=x^3$, $g:g(x)=\dfrac{1}{x}$ & $h:h(x)=x^2$

1. Is the domain of each of the above functions centered at origin? Justify.
   …………………………………………………………………………………………
2. Work out the following:
   - ***a.*** $f(-x)=$............................
   - ***b.*** $g(-x)=$............................
   - ***c.*** $h(-x)=$............................
3. Compare $f(-x), g(-x)$ & $h(-x)$ with $f(x), g(x)$ & $h(x)$ respectively.
   ……………………… ……………………… ………………………
4. Deduce the parity of each of the given function:
   …………………………………………………………………………………………
   …………………………………………………………………………………………
5. Find :
   - ***a.*** $f(x)+g(x)$ …………
   - ***b.*** $f(x)-g(x)$ …………
   - ***c.*** $f(x).g(x)$ …………
   - ***d.*** $f(x)+h(x)$ …………
   - ***e.*** $\dfrac{f(x)}{h(x)}$
   - ***f.*** $\dfrac{f(x)}{g(x)}$ …………
6. Use the above example to complete: If f & g are odd functions then:

Conclusions	*If*	*Then*	*Example*
	f *is odd* & $k\in\Re^*$	$k\ f$ is …………	$-2x^3$
	f & g *are odd functions*	………… $f+g$ is …………	……………..
	f & g *are* …………	………… $f.g$ *is odd* .	$x^3\cdot\left(\dfrac{1}{x}\right)$
	f is ……. & g *is* ……...		$\dfrac{x^3}{x^5}$
	f is odd, and h is even	The sum $f+h$ *is* ……...……………..	……………..
		The quotient is ……………………..	……………..

Consider the following points:



1- Determine the coordinates of the given points.

……….. ……….. ………..

……….. ……….. ………..

2- Compare and complete:

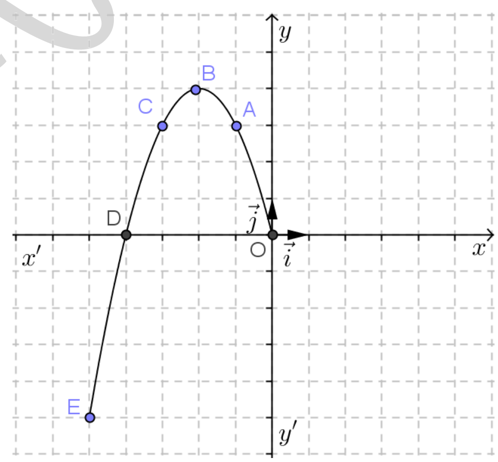
Points	Positions	Coordinates	
$A \& B$			
$C \& D$			
$E \& F$			
$(x; f(x)) \& (-x; -f(-x))$			

3- How can you tell that two points are symmetric with respect to origin?

…………………………………………………………………………………………

✓ *Graphical Study*:

Consider a function f defined by a part of its graph (C_f) where the points $A, B \& C$ belong to (C_f)



1- Complete the ***symmetry*** with respect to $O(0;0)$ of the points:

$A(x= \quad ; f(x)= \quad) \xrightarrow{symm.w.r.t\, origin} A'(-x= \quad ; f(-x)= \quad)$

$B(x= \quad ; f(x)= \quad) \xrightarrow{symm.w.r.t\, origin} B'(-x= \quad ; f(-x)= \quad)$

$C(x= \quad ; f(x)= \quad) \xrightarrow{symm.w.r.t\, origin} C'(-x= \quad ; f(-x)= \quad)$

$D(x= \quad ; f(x)= \quad) \xrightarrow{symm.w.r.t\, origin} D'(-x= \quad ; f(-x)= \quad)$

$E(x= \quad ; f(x)= \quad) \xrightarrow{symm.w.r.t\, origin} E'(-x= \quad ; f(-x)= \quad)$

2- Complete (C_f) by plotting the above points.

3- Suppose that f is defined by: $f(x) = x^3 - 4x$.

a. Study the parity of f.

………………………………… …………………………………

………………………………… …………………………………

………………………………… …………………………………

b. Is (C_f) symmetric with respect to origin? . . . .

4- What do you conclude? . . . .

The graph of an odd function is symmetric about *origin* $O(0;0)$

The graph of an even function is symmetric about $y'oy$