1- Consider a semi-circle (c) of center O and diameter $AB = 6cm$. Let M and N be two points on (c) where $B\hat{A}N = A\hat{B}M = 30^\circ$.

a. *Draw* figure.
b. *Prove* that the quadrilateral $ABNM$ is an isosceles trapezoid.
c. *Compute* the perimeter of $ABNM$.
d. The straight lines holding [AM] and [BN] intersect at a point I. *What* is the nature of triangle IAB?
e. Let P be the midpoint of [MN]. *Calculate* the lengths of IP, OP and AP. (***Ex-2ggb.***)

2- Given a circle $C(O;r)$ with two perpendicular diameters $[AB]$ and $[CD]$, let M be a point on the arc $\overset{\frown}{AC}$ through which a tangent is drawn cutting (CD) at E and (AB) at F.

a. *Sketch* a graph that translates the above text.
b. *Show* in two different methods that measure of $M\hat{E}O = 2M\hat{B}O$.
c. In what follows take $E\hat{F}O = 30^\circ$.
   i. *Compute* in terms of r the measure of $[MF]$.
   ii. *Deduce* also in terms of r the measure of $[EF]$.(***Ex-1ggb.***)

3- Consider a circle (λ) of center O, radius R and diameter $[AB]$. M is a point of (λ) and (d) is a tangent to (λ) at M. The tangents at A and B to (λ) intersects (d) at E and F respectively.

1) *What* is the nature of the quadrilateral $EABF$? Justify your answer.
2) *Find* the nature of triangle EOF.
3) *Prove* that the circle of center G and diameter $[EF]$ is tangent to (AB) at O.
4) (OE) intersects $[MA]$ at I and (OF) intersects $[MB]$ at J.
   a. *Demonstrate* that the straight line holding $[IJ]$ is parallel to (AB) and that $IJ = R$.
   b. *What* is the nature of quadrilateral $MJOI$?
5) In what follows, suppose that $R = 4cm$, $BF = 2cm$ & $AE = 2x$.
   a. *Calculate* in terms of x, the perimeter of $EABF$.
   b. *Compute* in terms of x, the area of the triangle EOF.
   c. *Calculate* in terms of x, the area of $EABF$.(***Ex-3ggb.***)

4- Let $[AB]$ be a fixed diameter of a circle $c(O;4.5cm)$ and M be a variable point on (c), distinct from points A & B.

a. *Show* that the sum $MA^2 + MB^2$ remains constant as M describes (c).
b. The medians $[AI]$ & $[BJ]$ intersect at G.
   Find locus of G as M moves on (c)? (***Ex-4ggb.***)

5- Given a circle (C) of center O and diameter $AB = 2R$. M is any point on (C). The circle of center M and radius $[MA]$ cuts (C) at E. D is the symmetric of A with respect to M.

a. *Show* that the points D, B and E are collinear.
b. *Compare* the measure of $[OM]$ and $[BD]$. *Deduce* the locus of D as M moves on (C). (***Ex-5ggb.***)

6- Let (λ) be a circle of center O and diameter $[PQ]$, T is a variable point belongs to (d), the tangent, to (λ) at P.

i. *Explain*, how to construct the second tangent issued from T to (λ), where R designates the second point of tangency.

ii. The straight line (TR) cuts the tangent drawn from Q at N; *verify* that: $TN = NQ + TP$.

iii. *What* is the nature of triangle TON.

iv. *Study* the locus of a point M the midpoint of [TQ] as T varies on (d). (***Ex-6ggb.***)

7- Given a semi-circle $c(O;r)$ and diameter $[AB]$, let (c') be another semi-circle of center O' and diameter $[AO]$. Now, consider a secant through A that cuts (c') at point C and (c) at point D.

a. *Outline* a figure.

b. *Show* that the triangles AOC and COD are congruent.

c. *Deduce* the relative position of point C with respect to $[AD]$.

d. *Prove* that, the tangents $(T)\&(\Delta)$ to (c') through point C and to (c) through point D respectively are parallel. (***Ex-7ggb.***)

8- Given a circle (C) of center O and diameter $AB = 6cm$, M is a variable point of (C), where D is the symmetric of A with respect to M. And the perpendicular to (AM) at A cuts (C) at E. Finally (DB) intersects (AE) at F.

a. *Draw* a figure.

b. *Find* the measure of $A\hat{M}B$ and prove that the triangle ABD is isosceles with principal vertex B.

c. *Prove* that $[ME]$ is a diameter for the given circle.

d. *Show* that (MO) is parallel to (BD), then *deduce* that E is the midpoint of $[AF]$.

e. *Confirm* that the triangle ABF is isosceles of vertex B.

f. *Find* the locus of D as M varies on (C).

g. If (DE) intersects (AB) at I.

i. *Show* that I is the center of gravity of the triangle ADF.

ii. *Deduce* that $IB = 2cm$. (***Ex-8ggb.***)

9- Consider a semi-circle (δ) of center O, radius $6cm$, and diameter $[AB]$. $[Ax)$ and $[By)$ are the tangents to (δ) at A and B respectively. C is a point on (δ) such that $C\hat{A}B = 30^\circ$. (AC) cuts $[By)$ at N and (BC) cuts $[Ax)$ at M. D and E are the midpoints of $[AM]$ and $[BN]$ respectively.

a. Prove that D, C and E are collinear and that (DE) is tangent to the semi circle at C.

b. Calculate the sides of the trapezoid $AMNB$.

10- Consider a circle (C) of center O, radius $3cm$, and diameter $[AB]$. Designate by (T) the tangent at A to (C) and by M any point on (T). From M, draw the other tangent (ME) which intersects (AB) at F. (OE) cuts (T) at G. S is the orthogonal projection of O on (GF).

a. *Draw* a sketch.

b. *Prove* that ΔAME is isosceles and that (MO) is the perpendicular bisector of $[AE]$.

c. *Show* that O is the orthocenter of ΔMGF. Deduce that points M, O & S are collinear.

d. Let I be the midpoint of $[AE]$. *Find* the locus of I as M varies on (T).

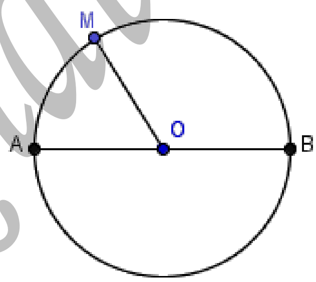
e. Suppose that $A\hat{M}E = 60^\circ$.

   i. *Find* the length of AM.

   ii. *Compute* the area of ΔAME. (***Ex-10ggb.***)

11- In the adjacent figure, $[AB]$ is a diameter of circle of center O and M is a point on circle.

1) *Draw* the tangent to the circle at M to meet the parallel drawn from point O to (AM) at point R.
2) *Show* that the triangle OMA is isosceles.
3) *Show* that $[OR]$ is the bisector of $M\hat{O}B$.
4) *Show* that the two triangles MOR and ROB are congruent.
5) *Deduce* that the points O, M, R and B belong to the same circle whose diameter is to be determined. (***Ex-11ggb.***)



12- Given a circle (c) of center O and radius R, $[BC]$ is a diameter of (c).
A is a point of (c) such that: mes $\widehat{AC} = 120^\circ$.

a. *Calculate* the angles of the triangle ABC.

b. *Deduce* that $AB = R$ and find the length of $[AC]$.

c. Through point C draw the tangent (T) to (c); (T) and (AB) intersect in point D.
Calculate interms of R, the length of CD and AD.

d. The line passing through O and parallel to (AB) cuts (CD) at E.
Show that the two triangles OAE and OCE are congruent.

e. *Deduce* that (AE) is tangent to (c) at A. (***Ex-12ggb.***)

13- Given the circle $C(O; 4\text{cm})$ and the line (d) that passes through O and cuts the circle (C) at A and B. Let M be the symmetric of O with respect to A. Draw (MT) tangent to (C) with tangency point T.

a. *Draw* a figure.

b. *Calculate* the length of: MT and AT.

c. *Calculate* the measure of angles: $A\hat{O}T$, $A\hat{M}T$ *and* $A\hat{B}T$, and then deduce the magnitude (norm) of $\overline{TB}$.

d. Line (d') is perpendicular to (d) at B & cuts tangent (MT) at E.
Show that quadrilateral $OBET$ is inscribed in a circle, whose center & radius are to be determined. (***Ex-13ggb.***)

14- Consider the triangle ABC to be inscribed in a circle $C(O; r\,cm)$, the heights ABC & ABC cut ABC at ABC & ABC respectively.

15- Consider a semi-circle $C(O;r)$, & diameter [*AB*]. The perpendicular bisector, (*d*), of [*OA*] cuts [*OA*] at *E* and (*C*) at *F*. *A* variable chord [*AM*] intersects [*EF*] at *K*. (***Ex-15ggb.***)

1) *Draw* figure.
2) *a- Show* that: *AF=R*.

   b- Calculate the area of triangle *AFB* interms of *R*.
3) *Prove* that quadrilateral *EKMB* is inscribed in a circle (*C′*) whose center *G* is to be determined.
4) Designate by *H* the foot of perpendicular drawn from *G* to (*AB*).
   a. *Calculate* the length of *BH* interms of *R*.
   b. *Determine* the locus of point *G* as *M* describes the arc $\widehat{BF}$ of the semi-circle (*C*).

16- Consider the circle $\eta(O;r)$, let (*d*) and (*d′*) be two parallel tangents to (η) at *A* and *B* respectively. (***Ex-16ggb.***)

a. *Construct* a figure.
b. *Confirm* that points *A*, *O* and *B* are collinear.
c. Let *C* be a point on (η), from *C* draw a tangent to (η) that intersects (*d*) at *A′* and (*d′*) at *B′*.
   i. *Prove* that: $A'\hat{O}B'$ is right.
   ii. *Prove* that: $A'B' = AA' + BB'$.
d. Let *I* and *J* be the respective midpoints of [*AC*] & [*BC*].
   i. *What* is the nature of quadrilateral *OICJ*?
   ii. The diagonals of *OICJ* intersect at a point *S*. *Find* the locus of *S* as *C* varies.

17- Consider the triangle *TON*, right at *O* where *OT* = 5*cm*, and $O\hat{T}N = 60^\circ$.

a. *Draw* the circle (*C*) of center *T* and tangent to (*ON*) at *O*.
b. The other tangent from *N* to (*C*) intersects it at *P*. *Calculate TN*, *ON*, and *PN*. Explain.
c. If [*TN*] intersects (*C*) at *E*, let *M* be the point diametrically opposite to *E* and let (*d*) be the tangent to (*C*) at *M*. *Prove* that lines (*d*) and (*OP*) are parallel.
d. *Show* that points *P*, *T*, *O* and *N* belong to the same circle (*C′*), whose center and radius are to be determined.
e. Let *B* be any variable point on (*C*) and *I* be the midpoint of [*BM*].
   Find the locus of *I* when *B* describes (*C*). (***Ex-17ggb.***)

18- Consider the circle $C(O; r = 4cm)$. Let (*d*) be a straight line passing through *O* and cuts (*C*) at *A* & *B*. Allow *M* to be the symmetric of *O* with respect to point *A*, and (*MT*) be a tangent drawn from *M* to (*C*) at point *T*.

a) *Draw* figure.
b) *Calculate* the measure of segments [*MT*] & [*AT*].
c) *Prove* that $T\hat{M}O = 30^\circ$.
d) Trace the line (*d′*) perpendicular to (*d*) at *B* cuts the (*MT*) at a point *E*.
   i. *Prove* that [*EO*) is the bisector of $T\hat{E}B$.
   ii. *Deduce* the nature of triangle *TBE*.
   iii. The interior bisectors of triangle *TBE* intersect at *J*. *Show* that *J* belongs to (*C*).

19- Given a circle (*C*) with center *O* and diameter $AB = 6cm$. Take a point *H* on [*OB*] such that $HB=1cm$. The perpendicular at *H* to (*OB*) cuts (*C*) in *P*. Let *C* be a variable point on arc $\overset{\frown}{AP}$ and *N* be the orthogonal projection of *B* on (*CP*).

a. *Draw* figure.

b. *Calculate* the magnitude of: *HP* and *BP*.

c. *Show* that: $N\hat{P}B = C\hat{A}B.$

d. 1- *Show* that the points *H*, *B*, *N* & *P* belong to the same circle.

2- *Deduce* that: $B\hat{H}N = B\hat{P}N = C\hat{A}B.$

e. The straight line (*HN*) cuts (*BC*) in *E*.

1. *Show* that (*HN*) is perpendicular to (*BC*).
2. *Find* the locus of point *E* as *C* varies on arc $\overset{\frown}{AP}$.

20- Consider a right angle $X\hat{O}Y, A$ is a fixed point on [*OX*) such that $OA = 4cm$. Let *I* be a variable point on the perpendicular at *A* to [*OX*) such that $IA>4cm$.
The circle $\eta(I;IA)$, cuts [*OY*) in *B* & *C* (*B* is between *O* & *C*). The straight line (*CI*) cuts the circle in point *P* and [*OX*) in point *K*.

1. *Show* that [*CA*) is the bisector of $B\hat{C}I$.
2. *What* does point *A* represent for arc $\overset{\frown}{BP}$.
3. *Show* that: $C\hat{P}A = A\hat{B}O.$
4. Let *H* be the orthogonal projection of *A* on (*CK*).
   a. *Show* that triangles *OAB* & *AHP* are congruent. Deduce that side $AH = 4cm$.
   b. *Find* the locus of *H* as *I* vary.
   c. *Show* that: $OC = HC$.
5. In what follows let the radius of $IA = 5cm$ & $OC = x$ ($x>5cm$).
   a. *Verify* that: $IH = x$-5.
   b. By using the right triangle *IAH*, *show* that *x* satisfies the equation:
   $(x-5)^2 = 9$. *Solve* it to find value of *x*.

21- Consider an isosceles triangle *ABC*, of main base $BC = 12cm$ and its height $AH = 8cm$. Let *O* be the point of [*BH*] such that $OB = 5cm$. The circle of center *O* and radius *OB* cuts (*AB*) in *M* and (*BC*) in *D*.

1. *a- Compare* $O\hat{M}B$ & $O\hat{B}M$.
   b- Show that $O\hat{M}B = A\hat{C}B.$
   c- Deduce that *OMAC* is an inscribed quadrilateral.
2. *a- Calculate* the measure of $\overline{AC}$.
   b- Show that triangles *BMD* & *AHC* are congruent.
   c- Deduce that $MD = 8cm$ and $MB = 6cm$.
3. (*MD*) cuts (*AH*) in *S*. Let $SH = x$.
   a. By using two congruent triangles of your choice, *show* that $SM = SH = x$.
   b. By using Pythagoras' theorem in triangle *HSD*, *show* that $x = 3cm$.
4. *a- Show* that *BHSM* is an inscribed quadrilateral.
   b- Determine its center *J* and calculate the length of its radius.

22- Consider a circle (*C*) of center *O* and diameter $AB = 6cm$. Let *M* be a variable point on (*C*). Designate by S the midpoint of arc *MB*. The straight line (*AS*) intersects (*MB*) & the tangent at *B* to (*C*) at *H* & *R* respectively. Finally (*AM*) intersects (*BS*) at *L*.

1. Draw figure.
2. a- *Show* that (*BS*) bisects angle $M\hat{B}R$.
   b- *Deduce* that triangle *HBR* is isosceles.
3. a- *What does H* represent for triangle *ALB*?
   b- *Deduce* that (*LH*) is parallel to (*BR*).
4. a- *Show* that triangles *SBR* & *SLH* are congruent.
   b- *Deduce* that quadrilateral *BRLH* is a rhombus.

23- Given the circle $C(O;5cm)$ of fixed diameter [*AB*]. Let (*xy*) be the tangent to (*C*) at *A*, and *M* be a variable point on (*C*). (*MP*) is the perpendicular to [*AB*] and (*MQ*) is the perpendicular to (*xy*). Let *I* be the midpoint of [*PQ*].

1. *Draw* a figure.
2. *Compare* [*AM*] & [*PQ*].
3. *Show* that the triangle *AIO* is right at *I*.
4. *Show* that [*MA*) is the bisector of $Q\hat{M}O$.
5. The tangent at *M* to (*C*) intersects (*xy*) at *T*. *Show* that (*TI*) is the perpendicular bisector of [*AM*]. *Deduce* that the points *T*, *I*, & *O* are collinear.
6. *G* is the center of gravity of triangle *AMB*. *Find* the locus of point *G*.

24- *C* is a point on a circle (φ) of center *O* and diameter $AB = 8cm$, such that $AC = 4cm$.

a. *Find* the nature of triangles *ACB* & *ACO*. Justify.
b. If *H* be the midpoint of [*CB*]. *Show* that (*OH*) is parallel to (*CA*). Compute *OH*.
c. Let be *I* the midpoint of [*AO*]; and the line (*CI*) intersects (φ) at point *D*.
   S*how* that the quadrilateral *CADO* is a rhombus.
d. *Prove* that points *D*, *O*, and *H* are collinear.
e. *Determine* the nature of triangle *CDB*.

25- Given *O* the midpoint of the segment $AB = 8cm$. Let *E* be a point on (*AB*) and exterior to [*AB*] such that $\frac{EA}{EB} = 3$.

1. *Prove* that $AE = 12cm$ & $BE = 4cm$ then verify that *B* is the midpoint of [*OE*].
2. *Draw* through *E* the perpendicular (*D*) to (*AB*) and designate by (*C*) the circle of diameter [*AB*]. A variable secant drawn through *A* cuts again the circle (*C*) at *I* and (*D*) at *M*. The perpendicular at *M* to (*D*) intersects (*OI*) at *O'*.
   a. *Draw* figure.
   b. *What* is the nature of triangle *AOI*?
   c. *Prove* that *O'IM* is an isosceles triangle of vertex *O'*.
3. Let *C'* be the circle of center *O'* and radius *O'I*.
   Prove that (*C'*) is tangent to (*D*) at *M* and to (*C*) at *I*.
4. Let *M'* be the symmetric of *M* with respect to *O'*.
   Prove that the points *M'*, *I*, and *B* are collinear.
5. *Prove* that points *B*, *E*, *M* and *I* belong to a circle whose center *K* is to be determined.
   Find locus of *K* as *I* describe the circle.