

**I-** a) Expand:

i-  $(2 - \sqrt{5})^2$ .

ii-  $(3 - \sqrt{2})^2$

b) Determine if possible, the roots of:

1)  $x^2 + (2 + \sqrt{5})x = -2\sqrt{5}$ .

2)  $x^2 + (3 + \sqrt{2})x + 3\sqrt{2} = 0$

**II-** Solve in  $\mathbb{R}$  the following equations:

a.  $9x^4 + 5x^2 - 4 = 0$

d.  $x - 2 + \sqrt{(3x - 4)^2} = 2$

b.  $(3x^2 - 2x + 1)^2 - 3(3x^2 - 2x + 1) + 2 = 0$

e.  $\sqrt{9x^2 - 6x + 1} = 3x - 1$

c.  $x(x + 1)(x^2 + x + 1) = 42$  (Let :  $u = x(x + 1)$ )

f.  $2x^6 - 3x^3 = -1$

**III-** Determine the numerical value of  $b$ , so that the equation:  $4x^2 + bx + b = 0$ , admits a double root in  $\mathbb{R}$ .

**IV-** Consider the trinomials:  $R(x) = 5x^2 + x - 6$  &  $N(x) = 7x^2 + 4x - 3$ .

a. For what values of  $x$  is:

i.  $R(x)$  strictly positive.

ii.  $N(x)$  strictly negative.

b. Deduce the solution of the system:  $\begin{cases} N(x) < 0 \\ R(x) > 0 \end{cases}$

c. Deduce the domain of:  $\sqrt{R(x)}$ .

**V-** Let  $p(x) = (m - 1)x^2 - 2(m + 1)x + 2m + 4$  be any parametric trinomial.

a. Study according to the values of  $m$  the existence of the roots of  $p(x)$ .

b. Determine the set of value of  $m$  so that  $p(x) > 0, \forall x \in \mathbb{R}$ .

c. Determine the numerical value of  $m$  if:

i. The roots of  $p(x)$  are opposite.

ii.  $+1$  is not a root of  $p(x)$ .

**VI-** Answer with justification by **true** or **false**:

a. The 2<sup>nd</sup> degree trinomial:  $E(x) = -3x^2 + 5x - 4$ , admits two strictly positive roots.

(Don't compute the roots).

b. If  $g(x) = ax^2 + bx + c$  and:

i.  $g(x) = 0$  admits two opposite real roots, then  $a \neq 0$  &  $c = 0$

ii. The product:  $ac > 0$ , then  $g(x)$  will always have two distinct roots.

iii.  $g(x) > 0$  for all real numbers  $x$ , then it is sufficient that  $\Delta < 0$ .

- VII-** Consider the parametric equation:  $(1 + m)x^2 + 3x + m = 0$ .
- 1) Determine the value of  $m$  in each of the following cases:
    - a.  $(E)$  admits a single root to be determined.
    - b.  $(E)$  has a double root to be determined.
  - 2) Given on the axis  $(x'ox)$  the points  $M'$  &  $M''$  respective abscissa  $x'$  &  $x''$ , and the point  $I(+1)$ .  
Find the values of  $m$  for which  $\overline{IM'} \times \overline{IM''} < 3$  ?
- VIII-** Let  $f(x) = (m + 1)x^2 - 2(m - 1)x - m - 5$  be a quadratic function with the parametric coefficient  $m$ .
- a. Prove that: For all  $m \in \mathbb{R} - \{-1\}$ ,  $f(x)$  admits in  $\mathbb{R}$  two distinct roots  $x'$  &  $x''$  to be determined.
  - b. Determine the set of values of  $m$  for which  $x' < x'' < 0$ .
- IX-** For what values of  $m$  is the inequality:  $(E): (3m - 1)x^2 - 2(3m - 1)x - 4 > 0$ .
- X-** Consider the parametric equation  $(E): (3m + 1)x^2 - 2(2m + 3)x + m - 3 = 0$ .
- a. Discuss according to the values of  $m$  the existence and the sign of the roots of  $(E)$ .
  - b. Determine  $m$  so that the sum of cube the roots of  $(E)$  equals zero.
- XI-** Consider the parametric equation  $(E): x^2 - 2x - m^2 + 2m - 5 = 0$
1. Show that: For all  $m \in \mathbb{R}$ ,  $(E)$  admits two different roots.
  2. Consider in the orthonormal system  $(O, \vec{i}, \vec{j})$  the points  $A(x', 0)$  &  $B(0, x'')$  where  $x'$  &  $x''$  are the roots of  $(E)$ .
    - a. Calculate  $m$  so that  $AB = \sqrt{20}cm$ .
    - b. Find  $m$  if area of the triangle  $AOB$  is  $6.5cm^2$
- XII-** Consider the parametric equation  $(E): (m + 1)x^2 - 2(m - 3)x + 2m - 5 = 0$ .
- a. Discuss according to the values of  $m$  the existence and the sign of the roots of  $(E)$ .
  - b. Determine the set of values of  $m$  such that  $(E)$  admits two distinct roots.
  - c. Determine  $m$  so that:  $(E) < 0, \forall x \in \mathbb{R}$ .
- XIII-** Consider the parametric equation  $(E): mx^2 - 2(m - 3)x + 2m - 1 = 0$ .
- a. Discuss according to the values of  $m$  the existence and the sign of the roots of  $(E)$ .
  - b. Find among the roots, when they exist, a relation independent of  $m$ .
- XIV-** Sara bought a number of scarfs at Malik's for 150\$. If each scarf had been 5\$ more, 5 fewer could have been purchased. Find the price of each scarf.

<b>Mastering problems</b>		
<i>Chapter</i>	<i>Exercises</i>	<i>Pages</i>
CH-: Second degree equations	1 & 2	3
	13, 14 & 16	5 & 6
	18 & 21	7 & 8