Al Mahdi High Schools	*Mathematics*	*10^{th}-Grade*
Name:	*" Generalities about Functions"*	*W.S-11*

I- The adjacent figure shows the representative curve, C_g of a function, $g : x^2 \to x^3 - 2x$ and a straight line (d).

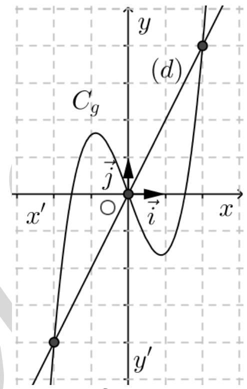


fig-1.

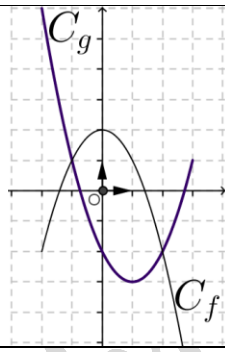
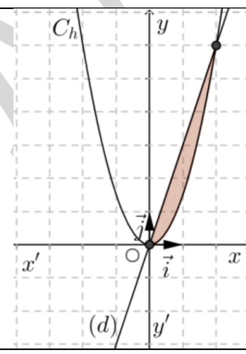
a. Find equation of straight line (d).

b. Solve the following algebraically:

- ***i.*** $x^3 - 2x = f(x)$.
- ***ii.*** $x^3 - 2x > f(x)$. .
- ***iii.*** $x^3 - 2x < f(x)$. .

c. What is the graphical interpretation for each of the above parts?

II- Complete the following table:

Figures		
Given	C_f & C_g are defined on $[-2;3]$	$h(x) = \frac{3}{2}x^2$ and $(d): -3x + y = 0$
Solve graphically	$f(x) = g(x)$.	$h(x) - 3x = 0$
	$f(x) - g(x) < 0$.	$h(x) < 3x$
	$f(x) - g(x) > 0$.	$h(x) \geq 3x$

III- Consider a function r defined over $\mathbb{R}$ by its representative curve C_r.

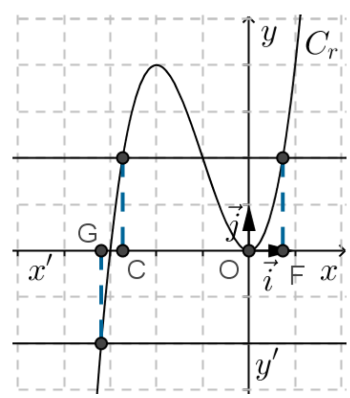


fig-3.

1- Determine graphically:

- ***i.*** $r(-2)$
- ***ii.*** The pre-image of 4 by r .

2- Solve graphically: $a)\ r(x) = 0$ $\quad c)\ r(x) = -2$

$b)\ r(x) = 4$ $\quad d)\ r(x) = -2$

3- Determine graphically the values of x for which:

$a)\ r(x) \leq 2$ $\quad c)\ r(x)$ is positive.

$b)\ r(x) + 2 \geq 0$ $\quad d)\ -3 \leq 2r(x) + 1 < 9$

4- Deduce the solution set of the inequality: $|r(x)| - 2 \leq 0$.

IV- Let h be a function defined over $\Re$ by: $h(x) = x^2 + 2x - 3$.

- ***a.*** Verify that h admits a minimum to be determined, and find the corresponding value of x at which this minimum is attained.
- ***b.*** Study variation of h over the intervals $]-\infty;-1]$ and $]-1;\infty]$.
- ***c.*** Find the roots of $x^2 + 2x - 3 = 0$, give the graphical meaning of the roots.
- ***d.*** Set up table of variations of h over its domain.
- ***e.*** Use above parts to trace the representative curve of h in an orthonormal system of axes.

V- Construct the representative curve C_f of the function f defined over the closed interval $[-3;7]$ such that:

- ➢ The image of 0 by f is 4.
- ➢ The pre images of 0 by f are 1 & 5.
- ➢ The adjacent table is the table of variation of f.

x	-3		-1		3		7
$f(x)$	4	↗	6	↘	-2	↗	1

VI- The (d) & C_h represents respectively a straight line $g(x)$ and the function h defined on $[0;5]$ by: $h(x) = ax^2 + bx + c$.

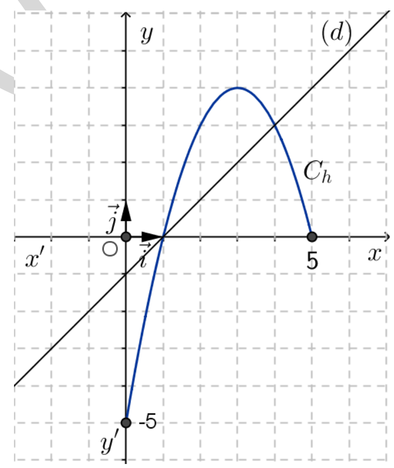


fig-4.

1) Use the C_h to show that: $a = -1, b = 6$ & $c = -5$.

- *a*. Prove by calculation, that $h(x) \leq 4$ for every $x \in [0;5]$.
- *b*. Deduce that h admits an extremum on [0,5] to be determined.
- *c*. Set up the table of variations of h.
- *d*. Compare without calculation $h(3.1)$ & $h(4.1)$, ***justify***.

2) Let k be another function defined by: $k(x) = \dfrac{\sqrt{h(x) - g(x)}}{h(x)}$.

- i. Study the relative positions of (d) & C_h.
- ii. Deduce the domain of definition of $k(x)$.

3) From now on, suppose that the given function h is defined on $[-5;5]$.

- a) Knowing that h is even, reproduce the above figure, then complete C_h on $[-5;5]$.
- b) Deduce the table of variations of h on $[-5;5]$.

VII- Draw the representative curve of a function h defined over the interval $[-4;8]$ by its table of signs and table of variation:

x	-4		-2		2		6		8
Sign of $h(x)$		-	0	+	0	-	0	+	

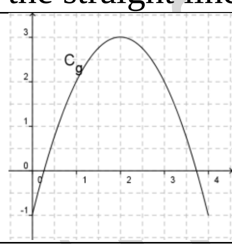
x	-4		-1		4		8
$h(x)$	-6	↗	3	↘	-2	↗	2

VIII- Let g be a function defined by its table of variation:

1) Determine the interval I over which g is defined, and then find the range.
2) Draw the representative curve of g over I.
3) Solve $g(x) \leq 0$ over I.
4) Show that $g(3) \times g(6) < 0$, interpret your result.

x	-1		1		3		6
$g(x)$	-3	↗	0	↘	-2	↗	1

IX- Choose with ***justification*** the only correct answer that corresponds to each question:

No.	Questions	Proposed choices		
		A	B	C
1.	The curve C_g of a function g defined by $g(x)=\frac{1+3x}{x}$ can be obtained from curve of the function $f(x)=\frac{1}{x}$ by:	A symmetry with respect to $y-axis$	A translation of vector $\vec{V}=3\vec{i}+0\vec{j}$	A translation of vector $\vec{V}=0\vec{i}+3\vec{j}$
2.	The function f defined over $]-\infty;-2[$ by $f(x)=\frac{1}{x-2}$ is	Strictly decreasing	Constant	Strictly increasing
4.	The graph of $h(x)=\sqrt{x+4}$ is the translate of the graph of $g(x)=\sqrt{x}$ by the translation:	$\vec{V}=-4\vec{i}$	$\vec{V}=2\vec{j}$	$\vec{V}=4\vec{i}$
5.	The graph of $r(x)=x^2+\sqrt{2}$ admits as an axis of symmetry the straight line:	$x=\sqrt{2}$	$x=0$	$x=-\sqrt{2}$
7.	C_g is the curve of a function g which is defined by $g(x)=$ and $D_g=$...... 	$g(x)=-(x+3)^2+2$ $D_g=[0;4]$	$g(x)=-(x-2)^2+3$ $D_g=[0;4]$	$g(x)=-(x+2)^2+3$ $D_g=]-1;4]$
8.	Consider f & g two functions defined by : $f(x)=x(x-4)$, and $g(x)=(x-2)^2$, then the curve (C_g) is the image of (C_f) by translation of vector $\vec{u}=$	$\vec{i}$	$4\vec{j}$	$-4\vec{j}$
9.	Consider f and g two functions defined by : $f(x)=\frac{1}{x}$, and $g(x)=\frac{x+1}{x}$ then the curve (C_g) is the image of (C_f) by translation of vector $\vec{u}=$	$\vec{u}=\vec{i}$	$\vec{u}=\vec{j}$	$\vec{u}=-\vec{j}$

X- Answer with ***justification*** by ***True*** or ***False***:

Let f be a function defined by its table of variation:

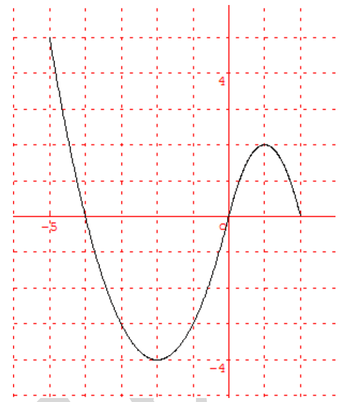
x	-4		-2		0		4		6
$f(x)$	-1	↗	4	↘	-3	↗	3	↘	1

i. f admits the interval $[-1;1]$ as a range.

ii. C_f is decreasing over the interval $[0;4]$.

iii. $f(1)<f(3)$

XI- Consider the function, f defined by its curve at the right.



Part-A: Graphical reading of (C_f)

1) Determine the domain of definition of f.
2) Study the variation of f.
3) Determine the maximum of f on $[0; 2]$.
4) Construct the table of variations of f.
   a- Solve graphically: $f(x) < 0, f(x) = -3$.
   b- Determine m so that $f(x) = m$ admits three distinct roots.
5) For $x \in [0; 2]$, we suppose that $f(x) = ax^2 + bx$.
   Verify that: $f(x) = -2x^2 + 4x$.

Part-B: consider the rectangle $EFGH$, such that $EF = x$ & $FG = -2x + 4$.

1)Determine as a function of x, the area $S(x)$ of rectangle $EFGH$.

2)Using part A, to determine the maximum value of $S(x)$

XII- Consider the table of variations of a function f :

x	-5		-1		1		3		5
$f(x)$	5	↘	0	↗	2	↘	0	↘	-1

a. Determine with justification:
- ***i.*** The interval over which f is defined.
- ***ii.*** The minimum of f over its domain.
- ***iii.*** Parity of f .

b. Determine $f(-1)$ and $f(1)$.

c. Compare $f(-2.5)$ & $f(-2)$. Justify your answer.

d. Let (C_f) be the representative curve of f .
- ***i.*** Construct on a reference frame (C_f).
- ***ii.*** Study the parity of f graphically.

e. Let g be a function defined by: $g(x) = \dfrac{x-1}{\sqrt{f(x)}}$, determine the domain of g .

XIII- Let f be a function defined by its table of variation:

x	-4		-2		0		4		6
$f(x)$	-1	↗	4	↘	-3	↗	3	↘	1

Answer with justification by ***True*** or ***False***:

a. $f(1) < f(3)$

b. $f(-2) \geq f(-1)$

c. $f(-3) < 4$

d. $f(5) = 0$

e. C_f cuts $x-axis$ over the interval $[0;4]$.

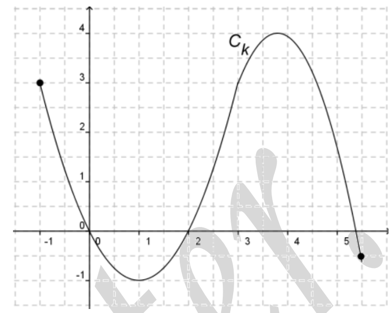
f. C_f cuts $x-axis$ over the interval $[4;6]$.

g. $f(-4) \times f(-2) < 0$

h. $f(x) > 0$ over the interval $[4;6]$.

XIV- Find values b and c so that the parabola with equation $y = 4x^2 - bx - c$ has a vertex at $(2,4)$?

XV- Let C_k be the representative curve of a function k.



- ***a.*** Determine the domain of definition of k.
- ***b.*** Set up the table of variation of k.
- ***c.*** Solve graphically:

  $1)\ k(x) = 0.$ $\quad$ $2)\ k(x) - 3 = 0$

  $3)\ k(x) > 0$ $\quad$ $4)\ k(x) < 0$

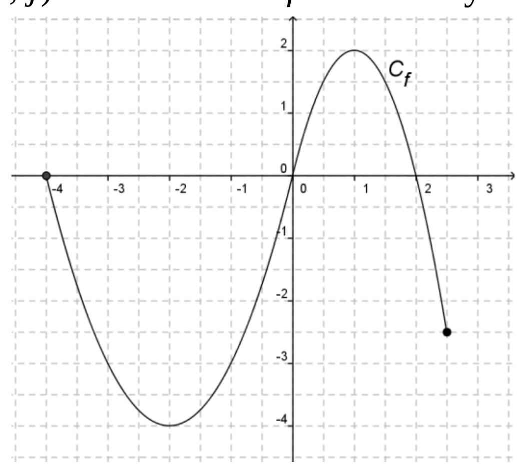
  $5)\ k(x) < 2$ $\quad$ $6)\ k(x) \geq 2$

  $7)\ 0 \leq k(x) \leq 1.$

XVI- f is a function defined on by: $f(x) = -x^2 - 2x + 4.$

- ***a.*** Verify that $f(x) - 5 = -(x+1)^2$.
  - ***i.*** Deduce that f admits a maximum to be determined.
  - ***ii.*** Find the value of x where this maximum is reached.
- ***b.*** Study the variation of f on each of the following intervals:
  - ***i.*** $I =]-\infty; -1].$
  - ***ii.*** $J = [-1; +\infty[$
- ***c.*** Set up the table of variation of f.
- ***d.*** Sketch the graph of f.
- ***e.*** Solve both ***algebraically*** and ***graphically*** the inequality: $f(x) \geq -2x.$

XVII- Consider in an orthonormal system of reference $(O, \vec{i}, \vec{j})$ the function f defined by its representative curve C_f.



- ***a.*** Precise the domain of f, deduce its parity.
- ***b.*** Set up table of variations of f.
- ***c.*** Solve graphically and interpret the results of:
  - ***i.*** $f(x) = 0$
  - ***ii.*** $f(x) > 0$
- ***d.*** Trace the straight line $(d): y = x$, then discuss graphically the inequality: $f(x) + x \leq 0$
- ***e.*** The part of the curve C_f over the interval $[-4;0]$, is the translate of the curve C_h of a function h given by $h(x) = x^2$ with a translation vector $\vec{V}$.
  - ***i.*** Determine coordinates of $\vec{V}$.
  - ***ii.*** Find equation of h over $[-4;0]$

XVIII- Let f be a function defined by $f(x)=x^2+2x-3$

a. Prove that f admits a minimum -4 at a point $A(x_A;-4)$ whose abscissa is to be determined.

b. Show that f is:

- ***i.*** Strictly decreasing on the interval $]-\infty;-1[$.
- ***ii.*** Strictly increasing on the interval $[-1;+\infty[$.

c. Set up the table of variations of f .

d. Construct the representative curve C_f of f in the orthonormal reference of axes $(O,\vec{i},\vec{j})$.

e. Solve algebraically then graphically:

- ***i.*** $f(x)=m$ such that $m\in\mathbb{R}$
- ***ii.*** $|f(x)|\leq 2$.

f. Construct the graphs of:

- ***i.*** $g(x)=f(x-1)$
- ***ii.*** $h(x)=f(x)+4$
- ***iii.*** $k(x)=|f(x)|$
- ***iv.*** $p(x)=f(|x|)$.

XIX- The adjacent graph shows the representative curve C_f of a function f defined over the closed interval $[-4;0]$ in an orthonormal system $(O,\vec{i},\vec{j})$.

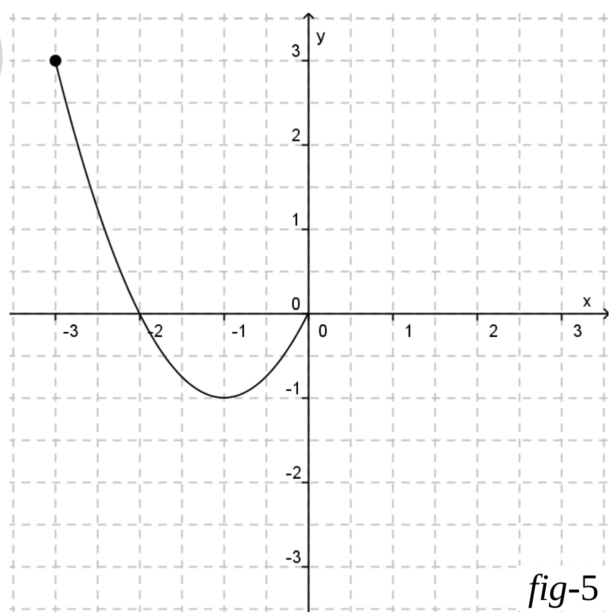


fig-5

a. Determine $f(0), f(-1)$ & $f(-3)$.

b. Compare graphically $f(-\sqrt{2})$ & $f(-\sqrt{3})$.

c. Solve graphically:

- ***i.*** $f(x)=0$
- ***ii.*** $f(x)<0$
- ***iii.*** $0<f(x)\leq 3$
- ***iv.*** $f(x)+3>-2x$

d. Set up the table of variations of f.

e. Find the numerical value s of a,b & c so that $f(x)=ax^2+bx+c$.

f. Let $r(x)=m$, discuss according to the values of m the number of solutions of the equation $r(x)=f(x)$

g. Let g be a function defined over $[-3;3]$ by $g(x)=x^2+2|x|$

- ***i.*** Detect the parity of g.
- ***ii.*** Write $g(x)$ without absolute value.
- ***iii.*** Trace the representative curve of g on the above system.

h. Let h be a function defined by $h(x)=f(-x)$.
Deduce the curve of h and its domain.

XX- Let f be a function defined by $f(x) = \frac{1}{2}x^2 - 2$ and (C) is its representative curve in an orthonormal reference$(O; \vec{i}; j)$.

1) Study the parity of f. Give a geometric interpretation.
2) a- Prove that f is increasing on$[0; +\infty[$. Deduce the variation of f on$]-\infty; 0]$.
   b- Prove that f admits a minimum on IR to be determined
   c- Complete the following table:

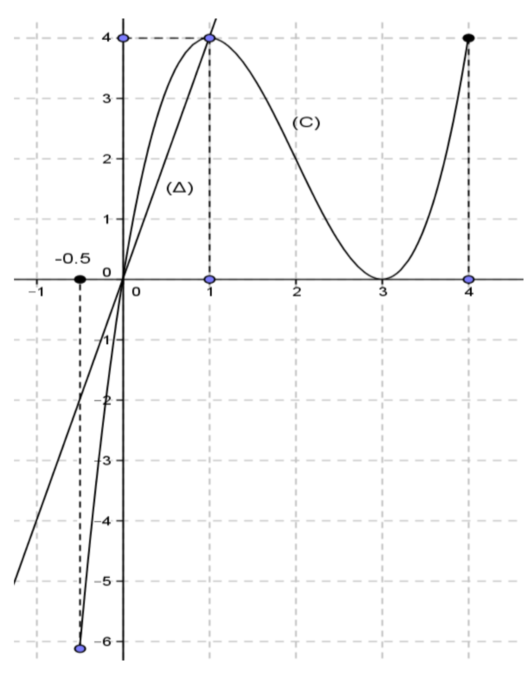
x	0	1	2	3	4
f(x)					

3) Trace (C).
4) Let $g(x) = x - 2$ and (d) is its curve.
   a- Trace (d) in the same system of axes as (C).
   b- Solve $f(x) = g(x)$.
   c- Solve graphically: $\frac{1}{2}x^2 < x$.

XXI- Let g be a function defined, on ℝ, by: g(x) = $(x-4)(x-2)$. Let (C_g) be the representative curve of g in an orthonormal system $(O; \vec{i}, \vec{j})$.

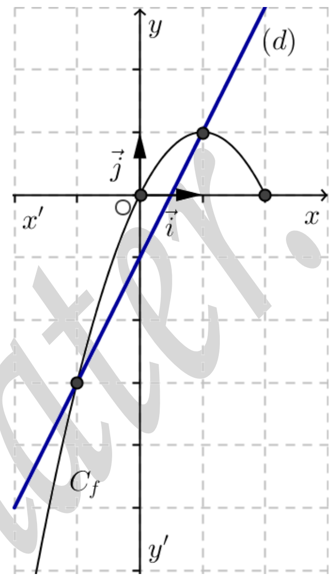
1) Find the common points between (C_g) and x'x.
2) Verify that g(x) = $(x - 3)^2 - 1$.
3) Prove that g is decreasing on $]-\infty, 3]$ and that g is increasing on $[3, +\infty[$.
4) Set up the table of variations of g, then deduce the minimum of g.
5) Let h be a function defined, on ℝ, by: h(x) = x^2.
   a- Draw (C_h), the representative curve of h, in an orthonormal system $(O; \vec{i}, \vec{j})$.
   b- Deduce the construction of (C_g) after translating (C_h) by a translation vector to be determined, then draw (C_g).

XXII- Consider in the adjacent figure a curve (C) that represents a function f and a line (Δ) defined by: $g(x) = 4x$.



1) Find the domain of definition of f.
2) Find f(1) and f(0) graphically.
3) Solve graphically f(x)= 4
4) Compare f(1.5) and f(2) .(without finding their values)
5) Find a local minimum and a local maximum of f.
6) Show that f is neither even nor odd over the interval [-1;1]
7) Setup the table of variation of f.
8) Solve graphically:
   a) $f(x) = g(x)$.
   b) $f(x) - 4x \leq 0$
   c) Deduce the domain of definition of the function
   d) r(x) = $\sqrt{4x - f(x)}$.

XXIII- Consider the function $f: x \longrightarrow ax^2 + bx$ and its curve C_f defined on the interval I.



Part-1: *Existence and parity*

- ***a.*** Find the interval I for which f admits an image.
- ***b.*** Discuss the parity of f.
- ***c.*** Utilize C_f to prove that: $a = -1 \& b = 2$.
- ***d.*** Justify that the standard form of f : $f(x) = -(x-1)^2 + 1$

Part-2: *Variation*

1) Prove that: $f(x) \leq 1$.
2) Deduce that f admits an extremum, and specify its nature.
3) Study variation of f over $]1,2]$ and set up table of variations of f.

Part-3: *Relative positions and translation*

- ***a)*** Use the graph to find equation of (d).
- ***b)*** Solve graphically:
  - $* \ f(x) > 0$
  - $* \ f(x) < 2x - 1$
- ***c)*** Let g be the image of f by the translation vector $\vec{S} = \vec{i} - \vec{j}$:
  - ***i.*** Determine the equation of $g(x)$.
  - ***ii.*** Trace on your answer sheet C_g.

XXIV- In an orthonormal system $(O; \vec{i}, \vec{j})$ consider the two vectors $\vec{S}(-x; 2x-3)$ and $\vec{N}(x; -1)$, where $x \in \mathbb{R}$.Let $f: x \mapsto f(x)$ be a function defined by $f(x) = \vec{S} \cdot \vec{N}$.

- ***a.*** Calculate $f(x)$ and find with justification the domain of f.
- ***b.*** Show that f admits a maximum 4 at a value of x to be determined.
- ***c.*** Study the variation of f over the intervals:
  - ***i.*** $]-\infty; -1]$.
  - ***ii.*** $[-1; +\infty[$.
- ***d.*** Set up table of variations of f, then draw C_f the representative curve of f.
- ***e.*** Determine graphically values of x for which vectors $\vec{S}$ & $\vec{N}$:
  - ***i.*** Are orthogonal.
  - ***ii.*** Form an acute angle.

Mastering problems		
Chapter	*Exercises*	*Pages*
CH-: Functions	6,7,9&12	349 to 351
	22	356
	23&24	357
	25	358