I- Consider a circle (C) of diameter[AB], center O and radius 4cm. let G be a point of [OB]. The perpendicular to (AB) at G cuts (C) in two points M and N. the line (MO) cuts again the circle (C) in point P.

a. Draw a figure.

b. Prove that the two triangles MGB and MAP are similar and deduce that: $MA \times MB = 8MG$.

c. Let E be the midpoint of [MA].

i. Prove that the two lines (OE) and (BM) are parallel.

ii. Find the locus of the point E as G moves on [OB].

II- Consider that the two circles $C(O;3cm) \& C'(O';6cm)$ are tangent externally at point A. Let B be a point of (C') diametrically opposite to A. The tangent to (C) , drawn from B, cuts (C') at D. Let I be the point of intersection of the drawn tangent with (C).

a. Draw a figure. Compute BI.

b. Compare the triangles ABD and BOI, then write ratio of similarity.

c. Calculate the measure of the sides AD and BD.

d. Calculate the area of triangle ABD, deduce area of triangle BOI.

III- Through a vertex of a parm $ABCD$, draw a secant to cut [BD] at point E and intersect the lines [BC] and (CD) at points F and G respectively.

a. Prove that triangles ADE and BEF are similar, and write the ratio of similitude.

b. Prove that triangles DEG and ABE are similar, and write the ratio of similitude.

c. Show that: $AE^2 = EF \times EG$.

d. Demonstrate that the product: $BF \times DG$ is constant.

IV- ABC is an isosceles triangle of vertex C inscribed in a circle (S). Through point C draw a ray [Cx) that intersects [AB] at D and (S) at E.
Show that: $AC^2 = CD \times CE$.

V- Consider the circle C(O; 4cm). [AE] is a fixed chord and H is a variable point on [AE]. The perpendicular bisector of [HE] cuts the circle in B and C and (AE) in F. The line (CH) cuts (AB) in K.

1. Find the nature of triangle CEH.

2. *a*- Show that: $C\hat{H}E = A\hat{B}C$.

b- Show that triangles CFH and CBK are similar. Write their ratio of similarity.

c- Deduce that: $\dfrac{CK}{CB} = \dfrac{CF}{CE}$.

3. Use the previous part to show that triangles BCE and CFK are similar.

VI- Given the two similar triangles $ABC \& DEF$ with $AB = 5cm, DE = 7cm$ and the area of triangle ABC is $12cm^2$. Find the area of the triangle DEF.

VII- Consider a triangle ABC such that: $AB = 2AC$. Let D be a point on [AC) where $AD = 2AB$

a. Prove that the two ABC & ABD are similar.

b. Show that $AB^2 = AC \times AD$.

c. Evaluate the ratio: $\dfrac{Area\, of\, \Delta ABD}{Area\, of\, \Delta ABC}$.

VIII- Draw a ray [Ax) on which we construct a semi-circle (c) of center O, diameter [AB] and radius r. I is a point of [Bx) such that $BI < r$. The tangent to (c) through point I intersects the circle at point M.

a. Prove that the triangles IBM & AMI are similar, deduce that $IM^2 = IB \times IA$.

b. The perpendicular drawn from O to (AB) meets (BM) at K and intersects (AM) at H. The straight-line (BH) cuts (AK) at J. Show that J belongs to (c).

i. Show that triangles BHO & AOK are similar and deduce the value of $OH \times OK$ in terms of $\boldsymbol{r}$.

ii. Deduce that $OM^2 = OH \times OK$.

c. Determine the locus of G the midpoint of [KB], as point I varies on [Bx).

IX- Consider a circle (C) of center O and radius R, and (D) is any line exterior to (C). Let M be a variable point on (D). Construct the rays [MA) and [MB) the two tangents drawn from M to (C). E is the orthogonal projection of O on (D). [AB] cuts (MO) in F and (OE) in I.

a. Determine the relative position of (OM) with respect to [AB].

b. Prove that the poi

c. nts M, E, I and F belong to the same circle whose diameter is to be determined.

d. Show that triangles AFO and AMO are similar and deduce that $OF \times OM = R^2$.

e. Determine the locus of F as M varies on (D).

X- Consider a circle (s) of center O with diameter $AB = 8cm$. The perpendicular at O to (AB) intersects (s) at point C. Let I be the midpoint of [OA]. The line (CI) cuts (s) at M.

1. Show that: $CI = 2\sqrt{5}$ & $CB = 4\sqrt{2}$.

2. a) Show that triangles CBI and AMI are similar and write their ratio of similarity.

b) Deduce that $MI = \dfrac{6\sqrt{5}}{5}$ & $AM = \dfrac{4\sqrt{10}}{5}$.

c) Verify that: $CM = \dfrac{16\sqrt{5}}{5}$.

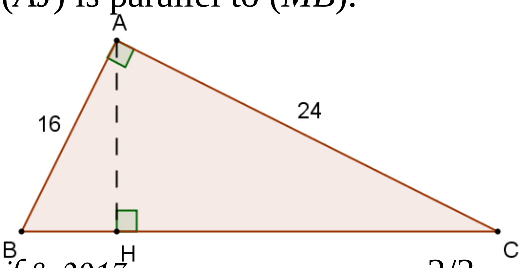
3. Let J be the midpoint of [CB].

a. Prove that $\dfrac{AM}{BJ} = \dfrac{CM}{AB}$.

b. Show that triangles ABJ and ACM are similar. Write homologous angles.

c. Compare the angles $A\hat{C}M$ & $A\hat{B}M$. Deduce that (AJ) is parallel to (MB).

XI- Refer to the figure below to find the ratio: $\dfrac{Area_{\Delta ABH}}{Area_{\Delta ABC}}$.



XII- In the figure below (c) is a circle of center O and diameter $AB = 12cm$. Let M be the midpoint of $[AO]$ and (r) be another circle of diameter [MO].

a. Trace on the same figure a line through M that intersects (c) in two points E & F and (r) at N. Designate by H the orthogonal projection of B on (EF).

b. Show that (ON) and (BH) are parallel.

c. If lines (BH) and (AN) intersect at K. Deduce that: $\dfrac{ON}{BK} = \dfrac{1}{2}$.

d. Verify that: $\dfrac{OM}{BO} = \dfrac{1}{2}$.

e. Show that the two triangles OMN and BOK are similar.

Deduce their equal angles.

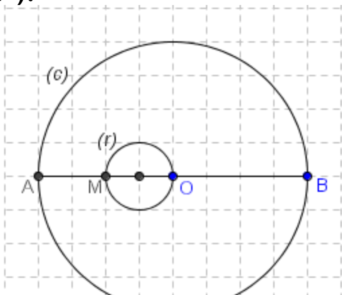
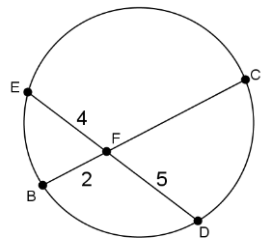


fig-1.

XIII-Let O be the midpoint of segment AB =12cm and D be a point on the perpendicular bisector of $[AB]$ such that OD = 3cm. C is the orthogonal projection of B on (AD).

1. Draw a figure to a real scale, and then compute the exact value of $[AD]$.
2. Prove that the triangles AOD & ACB are similar. Deduce the length $[AC]$ & $[BC]$.
3. Consider E to be the midpoint of $[AC]$, and (S) be the circle of center O and radius OE.
   a. Show that (AC) is the tangent to (S) at E.
   b. Calculate the radius of (S). Show that $\vec{OE} = \dfrac{1}{2}\vec{BC}$.

XIV-Refer to the figure to find the ratio: $\dfrac{Area_{\Delta BEF}}{Area_{\Delta CDF}}$.



XV- Find area of the triangle ABC, if the points $M, N \& P$ represent the respective midpoints of $[AB],[AC]\&[BC]$ and area of triangle MNP is $50cm^2$.