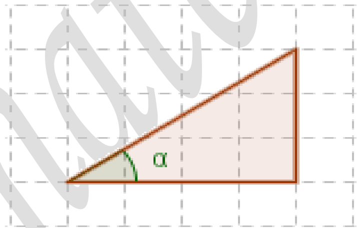
I- Consider the expression: $A=\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)^2+\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)^2$

a. Show that $A=1$.

b. Suppose that $\sin x=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ where x is an acute angle. Deduce the value of the corresponding co function $(\cos x)$.

c. Prove that $\tan x=2-\sqrt{3}$.

II- Consider the adjacent right triangle:



a. Prove the following identity: $1+\tan^2\alpha=\dfrac{1}{\cos^2\alpha}$.

b. Assume that $\tan\alpha=\dfrac{3}{4}$, then compute by two different ways the value of $\cos\alpha$.

III- RNK is a semi-equilateral triangle right at N, such that $N\hat{R}K=60^o$ and $RN=\sqrt{3}$.

a. Show that the exact value of RK is $2\sqrt{3}$, give its approximate value to the nearest *millimeter* by default.

b. Compute the exact value of the segment NK . (Use two different ways).

IV- Consider the expression: $R=\dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$ *and* $N=\dfrac{1}{1-2\sin^2\alpha}$.

a. Prove that: $R=N$.

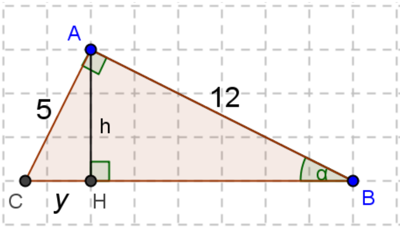
b. Deduce, the value of $\sin\alpha$ if $\tan\alpha=2$.

V- The following parts are independent"

a. Construct the equation of line (d) forming an angle α with the positive $x-axis$ and passing through the point $A(-1;4)$.

b. In an orthonormal system of axes, let $M(m-1,n+2)$ be any point on the straight line $(D): y=3x-11$, which makes an acute angle α with the positive $x-axis$ such that $\tan\alpha=m+2n$. Determine the values of m & n.

VI- Answer in the following order:



a. Find numerical value of α , to the nearest degree.

b. Determine the exact measure of side the BC.

c. Compute the exact area of the triangle ABC

d. <u>Without finding h,</u> use $\cos\alpha$ to deduce the numerical value of y .

VII- In the right triangle ABC of hypotenuse BC and $\sin\hat{B}=0.8$.

a. Calculate $\cos\hat{B}$ & $\tan\hat{B}$.

b. If $BC = 15cm$, then find the length of $AB \& AC$.

VIII- Consider the system: $\begin{cases} 6\cos\alpha - 4\sqrt{2}\cos\beta = -1 \\ 2\cos\alpha + \sqrt{2}\cos\beta = 2 \end{cases}$

a) Find the value of $\cos\alpha$ and $\cos\beta$.

b) Deduce the values of the acute angles α *and* β.

IX- Consider a triangle IJK right at K and that $J\hat{I}K = 15^o$.

a. Calculate the expression $A = \left(2+\sqrt{3}\right)\times\left(2-\sqrt{3}\right)$

b. Find the exact value of angle $I\hat{J}K$.

c. Knowing that $\tan\hat{I} = 2-\sqrt{3}$ & $A = 1$ Deduce the exact value of $\tan\hat{J}$.

X- In a triangle ABC right at A.

i. Compare: $\cos\hat{B}$ & $\sin\hat{C}$.

$\sin\hat{B}$ & $\cos\hat{C}$.

ii. Reduce: $a = \cos 10^o - \sin 80^o$.

$b = \sin^2 15 + \sin^2 75 = 1$.

$c = \left(\cos 35^o - \sin 55^o\right) + \left(\sin 35^o - \cos 55^o\right)$.

XI- Prove the following identities:

a. $(\sin x + \cos x)^2 - (\sin x - \cos x)^2 = 4\sin x.\cos x$.

b. $\cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$.

c. $\cos^4 a - \sin^4 a = \cos^2 a - \sin^2 a$.

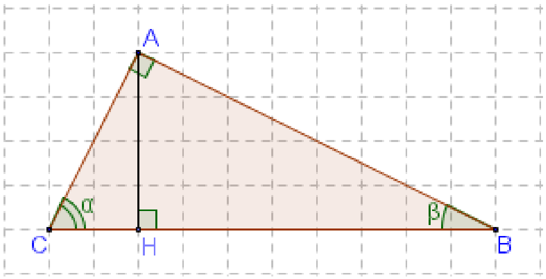
d. $\sin^4 a + \cos^4 a = 1 - 2\sin^2 a.\cos^2 a$.

e. $\dfrac{1-\sin y}{\cos y} = \dfrac{\cos y}{1+\sin y}$.

f. $\sin^4 a - \cos^4 a + 2\cos^2 a = 1$.

g. $\dfrac{1-\tan^2\alpha}{1+\tan^2\alpha} = 2\cos^2\alpha - 1$.

XII- Consider the following figure:



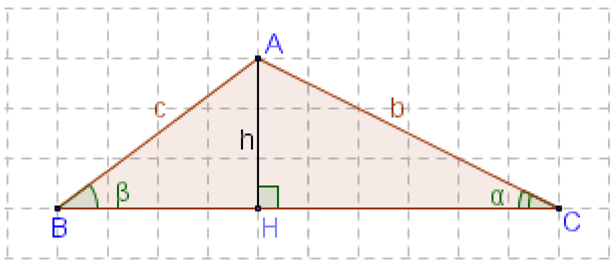
a. Find $\cos\alpha$ & $\cos\beta$ in 2 different triangles.

Deduce that $\begin{cases} AC^2 = CH \times CB. \\ AB^2 = BH \times BC. \end{cases}$

b. Prove that $\alpha = H\hat{A}B$ and $\beta = C\hat{A}H$

c. Find $\tan\alpha$ in 2 different triangles to show that $AH^2 = HB \times HC$.

XIII- Consider the triangle ABC:



a. Prove that $h = b\sin\alpha$.

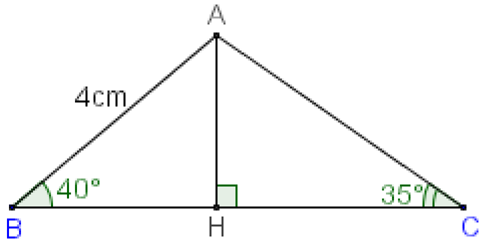
b. Find $\sin\beta$ in terms of h.

c. Verify that $\dfrac{c}{\sin\alpha} = \dfrac{b}{\sin\beta}$.

XIV- Given triangle ABC such that:

$AB = 4cm, \hat{B} = 40^o \; and \; \hat{C} = 35^o$; where $[AH]$ is a height relative to $[BC]$.

1) Compute the measure of all missing lengths.
2) Calculate perimeter and area of triangle *ABC*.



XV- α is the measure of an acute angle such that $\cos\alpha$ = **m**.

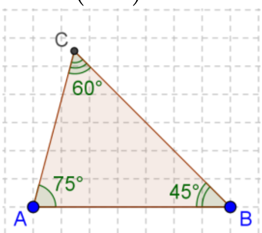
1. **Justify** which of the following propositions below is correct about **m**:

   a) $m = \frac{4\sqrt{2}}{9}$ b) $m = -\frac{4\sqrt{2}}{9}$ c) $m = \frac{9}{4\sqrt{2}}$

2. Deduce the numerical values of sin(α) and tan(α).
3. Without using the calculator, show that: A=$\frac{2(\cos 60^\circ + \sin 45^\circ)}{\tan 68^\circ \times \tan 22^\circ} \times \left(\sqrt{6} - \sqrt{3}\right)$ and B=$\frac{\sqrt{3}}{3}$ are reciprocals

XVI- The adjacent figure, **We suppose that:** $a = \cos\hat{A}$, $b = \cos\hat{B}$, $c = \cos\hat{C}$ and $\cos\left(75^0\right) = x$.

And we admit that *a*, *b* and *c* verify the relation: $a^2 + b^2 + c^2 + 2abc = 1$.



1) Show that: $4x^2 + 2x\sqrt{2} - 1 = 0$.
2) Consider the real numbers: $U = \frac{\sqrt{6} - \sqrt{2}}{4}$ and $V = \frac{-\sqrt{6} - \sqrt{2}}{4}$.

   a. Verify that U and V **are two solutions of the equation:** $4x^2 + 2x\sqrt{2} - 1 = 0$.

   b. Deduce the **exact value of** $\cos\left(75^0\right) = x$ and then deduce sin(75^0) and sin(15^0).

3) a) Compare the numbers: $\frac{\sqrt{6} + \sqrt{2}}{4}$ and $\frac{\sqrt{2 + \sqrt{3}}}{2}$.

   b) Deduce that: $\tan(75^0) = 2 + \sqrt{3}$, and then calculate $\cot(75^0)$.