1. Consider the following numerical expressions:

$A=\left(7+\frac{1}{7}\right)\div 5$; $B=0.0001407.$

a) Put the number A in the form of $\frac{a}{b}\left(where\, b\neq 0\right)$ list ***all*** steps of calculations.

b) Set B in the form of $c\times 10^{n}$, c being a number between 1 and 9, and n is an integer.

2. Perform the following numerical problems:

a) Prove that the product $N=-12^{100}\times\left(-\frac{2}{3}\right)^{-50}\times\left(\frac{1}{6}\right)^{149}$ is an ***integer***.

b) Express $S=\left(\frac{8}{5}\right)^{21}\times\left(\frac{5}{8}\right)^{20}$ in ***scientific notation***.

3. In the orthonormal system of axes $\left(x'Ox;y'Oy\right)$, consider the points $A\left(a,y_A\right)$, $B\left(b,y_B\right)$, $C\left(x_C,c\right)$ and $D\left(x_D,d\right)$, so that: $a=\frac{1-\frac{1}{6}}{1+\frac{1}{6}}; b=\frac{\left(2\times 10^{6}\right)^{2}\times 12.5}{7000\times 10^{10}};$

$c=\frac{4+51}{22}-\frac{14}{9}\times\frac{27}{21}+\frac{12\times 15}{40}\div 9$ and $d=10^{3}\times\left(\frac{-5}{2}\right)^{-2}\times\frac{5^{-1}}{32}$.

a) Specify the condition for which $\left(AB\right)$ is parallel to the ordinate axis? Is $\left(AB\right)\|y-axis$?

b) Specify the condition for which $\left(CD\right)$ is parallel to the abscissa axis? Is $\left(CD\right)\|x-axis$?

4. Consider in an orthonormal system of axes the points $R\left(r,s\right)$, $N\left(n,k\right)$and $K\left(3,7\right)$, so that: $s=\frac{8}{3}+5\div\left(1-\frac{2}{5}\right); k=\frac{55\times 10^{3}\times 2^{10}}{10^{4}\times 2^{9}}$ $n=\frac{2-\frac{1}{2}}{\left(\frac{1}{2}\right)^{2}}$ & $r=\frac{-2\times 10^{-2}\times\left(-6\right)\times 10^{7}}{4\times 10^{5}}$

a) Show that $\left(RN\right)$is parallel to one of the coordinate axes. Specify it.

b) Compare the coordinate of the points K & R to determine the nature of the triangle RNK.

i. Determine the center and the radius of the circle circumscribed bout triangle RNK.

ii. Justify that the point $S\left(7,9\right)$belongs to this circle.

5. Consider the numbers: $A=\frac{117}{63}$ *and* $B=-\frac{8}{7}$.

a) Show that A is ***reducible*** fraction.

b) Verify that A-B is a ***natural number***.

6. Answer the following:
   a) Write $C=\dfrac{10^{-4}\times 4\times 10^{6}\times 5^{2}}{2\times 10^{-10}}$ in simplest form possible.
   b) Arrange $\dfrac{11}{12};\dfrac{7}{8};\dfrac{5}{6}\ \&\ \dfrac{3}{4}$. in ascending order (***Note***: reduce to same denominator).
   c) Give the ***irreducible*** form of $4.25; 3.\overline{6}\ \&\ 6.\overline{15}$.
   d) How can you prove that the numbers 65 and 42 are relatively prime (***coprime***)
7. Given that: $A=\dfrac{9}{5}-\dfrac{3}{5}\times\dfrac{7}{9}$ *and* $B=\left(3-\dfrac{2}{3}\right)\div\dfrac{1}{9}$.
   a) Prove that *B* is a natural integer.
   b) If *B* is the number of number of boys in a class of 25 students, then find the percentage of girls in this class.
   c) Express *A*, in the form of an irreducible fraction.
   d) Find 25% of A.
8. Write the following in the simplest form possible:

$$R=\frac{\frac{5}{3}-1}{1-\frac{1}{6}} \quad ; \quad S=\frac{5}{2}-\frac{9}{2}\times\frac{1}{3} \quad ; \quad C=\frac{4\times 10^{12}\times 1.5}{9\times 10^{11}}.$$

9. Consider the following numbers:

$$R=\left(\frac{7}{4}-\frac{3}{2}\right)\div\left(\frac{3}{4}\times\frac{4}{9}-2\right) \qquad N=\frac{0.3\times 10^{-3}\times 0.006\times 10^{6}}{0.9\times\left(10^{2}\right)^{4}} \qquad S=\frac{5.1\times 10^{2}-270\times 10^{-1}}{4.83\times 10^{2}}$$

   By showing ***key*** steps of calculations:
   a) Express *R* in the form of an irreducible fraction and then in scientific notation.
   b) Prove that *N* is a ***decimal number***.
   c) Show that *S* is a ***natural number***.
10. Consider the numbers:

$$R=\left(\frac{2}{5}\right)^{2}-\frac{1}{5}; \qquad S=\frac{+1.25\times 10^{7}\times 8\times 10^{-4}}{4\times 10^{2}}; \qquad K=\frac{12}{40}\times\frac{15}{36}\div\frac{25}{8}.$$

   a) Find the values of *R*, *S* and *K* in details and give the results in simplest form.
   b) Which of the numbers *R*, *S* and *K* are opposite numbers and which of them are ***reciprocal*** of each other.
11. Consider the values: $A=\dfrac{7\times\left(10^{4}\right)^{3}}{5\times 10^{8}\times 2\times 10^{3}}$ *and* $B=\dfrac{2+\frac{1}{3}}{1-\frac{2}{3}}$. Prove that $A\times\dfrac{1}{B}=1$.
12. Consider the expression $A(x)=ax+1$,
   a) Compare $A(\sqrt{2})\ \&\ A(5)$ where $a>1$
   b) Prove that $A(-2)<A(-5)$ where $a<-1$

13. For what values of the variable n is the equation (E): $\frac{4^4-2^3}{2^3}=2^{2n+1}-1$ valid?

14. Let $A=\frac{7}{18}\times\frac{2}{7}-\left(\frac{5}{3}-1\right)^2$ and $B=\frac{0.3\times10^{-6}\times0.006\times10^{6}}{0.9\times\left(10^2\right)^4}$.

By writing all the steps of computations:

i. Express *A* in the form of an irreducible fraction.

ii. Script *B* in the form of $a\times10^n$ where a is a natural number.

15. Single out with ***justification***, the only correct answer for each of the following questions:

No.	***Questions***	***Expected answers***		
		a	***b***	***c***
1.	$2\pi,-\sqrt{39},\sqrt{10^{10}},\frac{\sqrt{9}}{3},0.\overline{09}.$ Among these numbers, there is	One irrational number	Two irrational numbers	Three irrational numbers
2.	$0.\overline{4}+2.\overline{14}$	$2.\overline{54}$	$2.\overline{58}$	$2.\overline{418}$
3.	$5^{11}-5^{10}=$	4×5^{10}.	5^{21}	5^{1}
4.	$2^2+2^{-2}=$	2^0	4^0	$\frac{17}{4}$
5.	$3\times10^{-2}+0.05=$	3.05×10^{-2}	$\frac{2}{25}$	0.35
6.	$\sqrt{0.\overline{4}}=$	$0.\overline{2}$	0.2	$\frac{2}{3}$
7.	$\frac{4^4-2^3}{2^3}=$	2^5-1	2^2	2^8
8.	$\sqrt{7.0\overline{1}-\frac{4}{15}}=$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{5\sqrt{29}}{5}$
9.	$3^{-1}-3^{-3}=$	$\left(\frac{2}{3}\right)^3$	$\frac{2^3}{3}$	$-\left(\frac{2}{3}\right)^3$
10.	$3^{12}+3^{12}+3^{12}=$	9^{12}	9^{36}	3^{13}

16. Consider the numerical expressions: $R=\left(2^n+2\right)^2-1$ and $S=\frac{4^n+4\cdot2^n+3}{4^n+2^n}$

a) Write R in an expanded form.

b) Show that: $S=\frac{2^n+3}{2^n}$.

c) Find the numerical value of n, so that $S=25$.

17. Given a rectangle $ABCD$ such that $AB = \left(4-\sqrt{2}\right)cm$ *and* $AD = \left(4+\sqrt{2}\right)cm.$

i. Decide which of the two given dimensions is the width of the rectangle.(Explain)

ii. Verify that $AC = 6.$

iii. Compute the side of a square having an area equivalent to the perimeter of rectangle $ABCD.$

18. Express the following numerical expressions in lowest terms possible where a & b are two real number different than zero:

$$A = \frac{(-3)^5(-12)^3(-4)^{10}}{(8)^4(24)^5} \; ; \; B = \frac{(-5)^3(-8)^3(-9)^2}{15^2.12^4}; \qquad C = \frac{a^{3n} - a^{n+2}}{a^{2n} - a^{n+1}}.$$

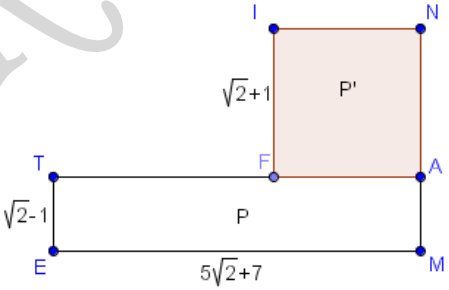
$$D = \frac{4^{n+2} - 4^n}{2^{n+2} - 2^{n-1}} \qquad ; \qquad E = \frac{\left(\frac{a^3.b}{a^2.b^5}\right)^{-2}}{\left(\frac{a^{-1}.b}{a^{-3}.b^4}\right)^3} \qquad ; \qquad F = \sqrt{\frac{8^{20} + 4^{10}}{8^4 + 4^{26}}}.$$

19. Consider the rectangle $MATE$, of dimensions $ET = \sqrt{2} - 1$ & $EM = 5\sqrt{2} + 7.$ and a square $AFIN$ of side $AN = \sqrt{2} + 1.$

i. Show that the two given plane geometrical shapes have equal areas.

ii. Prove that $MF = \sqrt{6}.$

iii. Denote by p *and* p' the respective perimeters of the given quadrilaterals. Confirm that $p = 3p'.$



20. Given that a, b & c are three real non – zero numbers, so that the ratio of 2 to a is equal to the sum of the reciprocals of b & c.

a) Convert the above statement into a mathematical equation in a, b & c

b) Calculate b, so that $a = -2$ & $c = -\frac{4}{3}.$

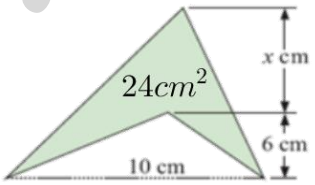
c) Is it possible to find the value of c such that $a = 1.5$ & $b = \frac{3}{4}$? Why?

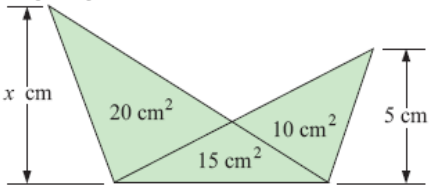
d) In this part, suppose that: $c = -(x+1)$ & $b = x - 1$, where $x > 0$.

i) Prove that: $\frac{1}{a} = \frac{1}{x^2 - 1}.$

ii) Calculate the numerical value of x, if $a = 3.$

21. Find the numerical value of x in each of the following figures:





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