Al- Mahdi High	Mathematics	11^{th}-Grade
Name:	Limits & Continuity	W.S-3

I- Choose with justification the only correct answer:

No.	Propositions	Expected answers		
		a	b	c
1.	$\lim\limits_{x\to 0^+} \dfrac{x+1}{x^2-x}$	$-\infty$	$+\infty$	$\pm\infty$
2.	$\lim\limits_{x\to -\infty} \dfrac{x}{\sqrt{x^2+4x}}$	$+1$	-1	0^-

II- Determine the following limits:

1) $\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^3+8}$

2) $\lim\limits_{x\to +\infty} \sqrt{x+1}-x$

3) $\lim\limits_{x\to +3} \dfrac{\sqrt{x+1}-2\sqrt{x-2}}{x-3}$

4) $\lim\limits_{\substack{x\to 3\\ x>3}} \dfrac{\sqrt{x+1}-2}{x-3}$

5) $\lim\limits_{x\to -3} \dfrac{x^2+6x+9}{x^2+4x+3}$

6) $\lim\limits_{x\to 1} \dfrac{x^3-x^2+x-1}{x-1}$

7) $\lim\limits_{x\to -\infty} \dfrac{(2x^3-5)^2}{(3x^2+1)^3}$

8) $\lim\limits_{x\to 0} \dfrac{|x|+x}{x}$

9) $\lim\limits_{x\to 0} \dfrac{|x^2-2x|+3x}{|x|}$

10) $\lim\limits_{x\to 0^-} \left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)$

III- Let r be a function defined by: $r(x)=\dfrac{\sqrt{x^3+1}-1}{x}$

a. Show that, for all $x\neq 0$, $r(x)=\dfrac{x^2}{\sqrt{x^3+1}+1}$, then deduce the limit of $r(x)$ as x tends to zero.

b. If $n(x)=\dfrac{\sqrt{x+3}-\sqrt{3}}{x}$, then determine $\lim\limits_{x\to 0} n(x)$

IV- 1) If $f(x)=\dfrac{\cos x}{x}$, then prove that: $-\dfrac{1}{x}\leq f(x)\leq \dfrac{1}{x}$, for all $x>0$.

2) Deduce: $\lim\limits_{x\to +\infty} \dfrac{\cos x + x}{x}$

V- Let f be the function defined by: $f(x)=\dfrac{5-3x}{2x+1}$

a. Determine the domain of definition of f.

b. Calculate the limits of f at the open boundaries of its domain.

c. Deduce asymptotes of f.

VI- Consider the rational functions defined respectively by: $f(x)=\frac{x+3}{2-x}$ and $g(x)=\frac{3x-2}{(x+3)^2}$

a. Calculate limits of f as x tends to: $\pm\infty \& 2$.

b. Deduce the asymptotes of curve of f.

c. Calculate limits of g as x tends to: $\pm\infty \& -3$.

d. Deduce the asymptotes of curve of g.

VII- Which of the following functions admit asymptotes that are parallel to either of the coordinate axes? Justify and write equation of the asymptotes.

a) $f(x)=\frac{-2x+1}{x+3}$

b) $g(x)=x^2+1$

c) $k(x)=\frac{x-5}{x^2-2}$

d) $h(x)=\frac{-2}{x-1}$

e) $s(x)=\frac{x^2-2x+3}{x-1}$

VIII- Let g be the function defined by: $g(x)=\frac{3x^2-2x-1}{x+2}$ and its representative curve C_g.

a. Determine the interval over which the function g admits a curve.

b. Determine the real numbers $a,b \& c$ so that, for all $x\neq -2$, $g(x)=ax+b+\frac{c}{x+2}$.

c. Does the function g admit a vertical asymptote? Find it.

d. Calculate the $\lim\limits_{x\to\pm\infty} g(x)$.

e. Calculate $\lim\limits_{x\to\pm\infty}[g(x)-(ax+b)]$. Interpret the result graphically.

f. Study over the domain of g, the relative position of C_g with respect to the straight line (d) of equation $y=ax+b$.

IX- Consider the function g defined by: $y=g(x)=\begin{cases}\frac{x^2+4x-5}{x-1} & x<-2\\ 2x+7 & -2\leq x<1\\ x^2-2a+1 & x\geq 1\end{cases}$

a. Determine the following limits of $g(x)$ as $x\to-\infty \& x\to 0$

b. Prove that the limit of g exists at $x=-2$

c. Find the numerical value of a, so that $\lim\limits_{x\to 1} g(x)$ exists.

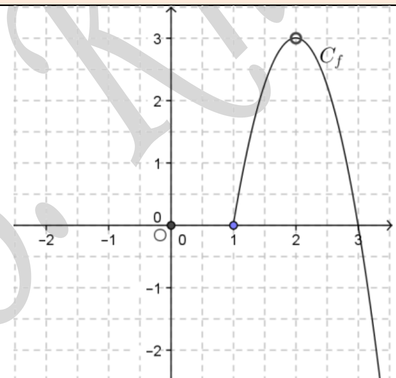
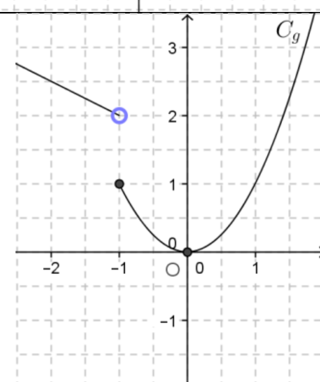
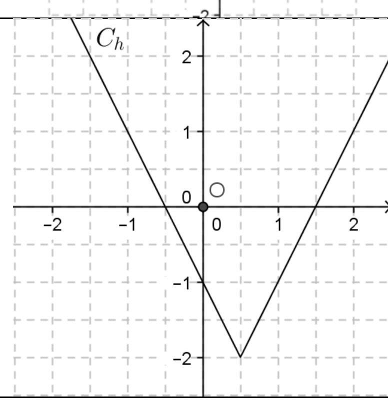
X- Find the value a if the limit of the given function exists:

$$f(x)=\begin{cases} x^2-2a & for\ x<2 \\ 3a+x & for\ x>2 \end{cases}$$

XI- Consider the function f defined by $f(x)=\begin{cases} x+2a-b & for\ x<1 \\ a+x+b-1 & for\ 1<x<3 \\ x^2-2a-5b & for\ x>3 \end{cases}$

Find the values of $a\ \&\ b$ if $\lim\limits_{x\to 1} f(x)\ \&\ \lim\limits_{x\to 3} f(x)$ exist.

XII- Answer the following Questions:

No.	*Questions*	*Curves*
A.	*i*- Determine $f(2)$ & $\lim\limits_{x\to +2} f(x)$. *ii*- Is f continuous at $x=2$?	
B.	1) Determine $g(-1)$ & $\lim\limits_{x\to -1} g(x)$. 2) Is g continuous at $x=-1$?	
C.	a- Determine $h(0.5)$ & $\lim\limits_{x\to +\frac{1}{2}} h(x)$. b- Is h continuous at $x=0.5$?	

XIII- Consider the function f defined by: $y = f(x) = \begin{cases} 2x^2+1 & x<0 \\ ax^2+3x+1 & 0 \le x < 3 \\ \dfrac{bx^2-7}{x^2+2x-1} & x>3 \\ c+3 & x=3 \end{cases}$

a. Prove that f is continuous at $x=0\ \forall a \in \mathbb{R}$.

b. Determine the value of b so that $y=-3$ is a horizontal asymptote of C_f at $+\infty$.

c. Find the values of a & c so that f is continuous at $x=3$.

XIV- Consider the curve of the real function f:

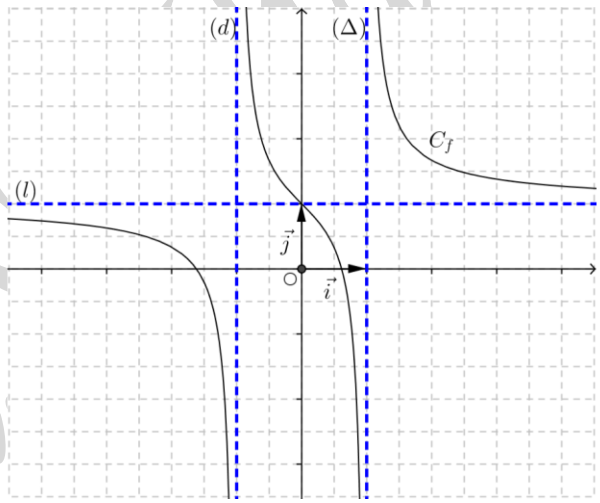
Determine graphically:

a) The domain of f.

b) The limits:

- $\lim\limits_{x\to-\infty} f(x)$ - $\lim\limits_{x\to+\infty} f(x)$
- $\lim\limits_{\substack{x\to1 \\ x<1}} f(x)$ - $\lim\limits_{\substack{x\to1 \\ x>1}} f(x)$
- $\lim\limits_{x\to-1} f(x)$

c) The equations of the given asymptotes.



XV- Consider the functions f & g defined by:

$f(x) = \begin{cases} \dfrac{ax^2-4x-12}{4-x^2} & x<-2 \\ -2x-2 & -2 \le x < 3 \\ x^2+b & x \ge 3 \end{cases}$	$g(x) = \begin{cases} \dfrac{x^2-x-2}{4-x^2} & x<-1 \\ 2x-1 & -1 \le x < 3 \\ x^2+c & x \ge 3 \end{cases}$
a) Find a so that $y=-1$ is an asymptote of C_f as $x\to-\infty$ b) If $a=1$, then study the continuity of f at $x=-2$. c) Find b so that f is continuous at $x=3$	1) Does g admit a limit at $x=-1$? 2) Determine the value of c so that the function g is continuous at $x=3$