1. Perform the following calculations:
   - ***a.*** Show that $a = \left(\sqrt{3}-1\right)^2 + \sqrt{12}$ is a natural number.
   - ***b.*** Prove that $b = \left(\sqrt{6}-\sqrt{11}\right)\left(\sqrt{6}+\sqrt{11}\right)$ is an integer.
   - ***c.*** Verify that $c = \dfrac{1}{2+\sqrt{7}} + \dfrac{1}{2-\sqrt{7}}$ is a rational number.
   - ***d.*** Confirm that $\sqrt{(\pi-5)^2} + 3\sqrt{7-2\sqrt{12}} \times \sqrt{7+2\sqrt{12}} + \sqrt{(\pi-1)^2}$ belongs to set $\mathbb{N}$.

2. Given: $X = -2\sqrt{7} - 3\sqrt{28} + 6\sqrt{63}$ *and* $Y = 2\sqrt{50} + \sqrt{72} - \sqrt{128}$.
   - ***a.*** Write X & Y in the form of $a\sqrt{b}$.
   - ***b.*** Compare X and Y.
   - ***c.*** Deduce if $X - Y > 0$.

3. $A = 7 + 4\sqrt{3}$ *and* $B = 7 - 4\sqrt{3}$. Prove that $\dfrac{A}{B} + \dfrac{B}{A}$ *and* $\sqrt{A}.\sqrt{B}$ numbers are integers:

4. Compare the numbers $3\sqrt{2}$ *and* $2\sqrt{5}$ then simplify $\sqrt{\left(3\sqrt{2}-2\sqrt{5}\right)^2}$.

5. Develop $\left(1-\sqrt{3}\right)^2$ then deduce another writing of $\sqrt{4-2\sqrt{3}}$ using only 1 radical sign.

6. Given: $R = \sqrt{7-4\sqrt{3}}$ *and* $N = \sqrt{9+4\sqrt{2}}$.
   - ***a.*** Write R and N in the form of one radical.
   - ***b.*** Rationalize the denominator of $\dfrac{R}{N}$.

7. Let $ab = 2\sqrt{3}$ *and* $a + b = 2 + 2\sqrt{3}$ be the product and sum of any two real numbers, find the numerical value of:
   - ***i.*** $a^2b + ab^2$.
   - ***ii.*** $a^2 + b^2$.
   - ***iii.*** $(a-2)(b-2)$.

8. Given the two real numbers a *and* b such that:
   $a + b = \sqrt{3+2\sqrt{2}}$ *and* $ab = \sqrt{3-2\sqrt{2}}$.
   Compute and simplify the following:
   - ***a.*** $\dfrac{1}{a} + \dfrac{1}{b}$.
   - ***b.*** $a^2b + ab^2$.

9. Given: $a = \dfrac{1}{2\sqrt{13}-6}$ *and* $b = \dfrac{1}{2\sqrt{13}+6}$. Calculate:
   - ***a.*** ab.
   - ***b.*** $a + b$
   - ***c.*** $a^2 - b^2$.

10. Calculate and simplify: $\dfrac{5\sqrt{2}}{2+\sqrt{2}} + \dfrac{10}{2-\sqrt{2}}$ *and* $\dfrac{\sqrt{8}+2}{\sqrt{5}+\sqrt{80}-\sqrt{45}}$.

11.Consider the expressions: $E=\sqrt{48}+\sqrt{20}$ *and* $F=\sqrt{108}-\sqrt{45}$.

a. *i*) Write E and F in the form $a\sqrt{3}+b\sqrt{5}$ where a & b are two integers.
ii) Verify that the product $E\times F$ is equal to a whole number.

b. Write the expression $G=\dfrac{\sqrt{48}+\sqrt{20}}{\sqrt{108}+\sqrt{45}}$ in the form of an irreducible fraction.

12.Indicate with justification, the only correct answer for each of the following questions:

No.	*Questions*	*Expected answers*		
		A	*B*	*C*
1.	The sign of $3\sqrt{2}-5$ is	Positive	Negative	Cannot decide
2.	If $a=-\dfrac{\sqrt{2+\sqrt{3}}}{2}$ and $b=\dfrac{\sqrt{2}+\sqrt{6}}{4}$, then	$a=b$	$a+b=0$	$a=2b$
3.	$\sqrt{(3.14-\pi)^2}=$	0	$\pi-3.14$	$3.14-\pi$
4.	$\sqrt{16+9}=$	7	12	5
5.	$\sqrt{\left(2-\sqrt{2}\right)^2}=$	$2-\sqrt{2}$	$2+\sqrt{2}$	$\sqrt{2}-2$
6.	If $A(z)=\sqrt{\dfrac{9}{z^2}+\dfrac{6}{z}+1}$ where $z+3<0$ then, $A(z)=$	$-\dfrac{z+3}{z}$	$\dfrac{z+3}{z}$	$\dfrac{3}{z}+1+\sqrt{\dfrac{6}{z}}$
7.	$\left[\sqrt{6}\times\sqrt{1-\dfrac{\sqrt{5}}{3}}\right]^2=$	$\sqrt{5}-\sqrt{6}$	$\left(\sqrt{5}-1\right)^2$	$\left(1+\sqrt{5}\right)^2$
8.	If $x<0$ then, $\sqrt{\dfrac{x^2}{4}+\dfrac{4x^2}{9}}=$	$\dfrac{5x}{6}$	$-\dfrac{5x}{6}$	$\pm\dfrac{5x}{6}$
9.	If $c\left(O;\sqrt{8}\right)$ & $c'\left(O';\sqrt{18}\right)$ are two circles, so that $OO'=\sqrt{50}$, then (c) & (c') are	Tangent externally	Intersecting	Tangent internally

13. Suppose that $A=(5+2\sqrt{7})^2$ *and* $B=(5-2\sqrt{7})^2$.

a. Expand then reduce the given expressions.

b. Use the previous results to compute the expressions: $E=\dfrac{53+20\sqrt{7}}{5+2\sqrt{7}}-\dfrac{53-20\sqrt{7}}{5-2\sqrt{7}}$.

c. Express the number $C=\sqrt{53-20\sqrt{7}}$ using only one radical sign.

14. Answer the following:

a. Calculate $(1-\sqrt{2})^2$ then prove that: $x=\sqrt{3-2\sqrt{2}}-\sqrt{3+2\sqrt{2}}$ is an integer.

b. Evaluate $(\sqrt{3}-\sqrt{5})^2$ then verify that: $(\sqrt{5}+\sqrt{3})\sqrt{8-2\sqrt{15}}$ is an integer.

c. Calculate $E=x^3+3x^2y+3xy^2+y^3$ *and* $F=(x+y)^3$ *for* $x=\sqrt{5}$ *and* $y=\sqrt{2}$.

d. Compare E and F.

15. a) Simplify $R=\sqrt{a^3+a^2-a-1}$ such that a is greater than 1.
b) Solve $R=0$.
c) Determine the numerical value of $R(2)$.

16. Carry out the following:

a. $\left(2\sqrt{5x}-\sqrt{15x-1}\right)\left(2\sqrt{5x}+\sqrt{15x-1}\right)$. Where x is a srtictly positive integer.

b. $\sqrt{75m-25}+\sqrt{108m-36}$. Where $m>1$

c. $\left(x-\dfrac{1-\sqrt{3}}{2}\right)\left(x+\dfrac{1-\sqrt{3}}{2}\right)$.

d. $\left(2\sqrt{3}-\sqrt{13}\right)^{51}\left(2\sqrt{3}+\sqrt{13}\right)^{51}$.

e. $N=\dfrac{a^2\left(\sqrt{3}+1\right)^2}{4}+\dfrac{a^2\left(\sqrt{3}-1\right)^2}{4}$. Deduce the value of $\sqrt{N}$, *where* $a\geq 0$.

17. Prove the following identities:

$\sqrt{4+\sqrt{7}}=\sqrt{\dfrac{7}{2}}+\sqrt{\dfrac{1}{2}}$ $\qquad$ $\sqrt{9+4\sqrt{5}}=2+\sqrt{5}$ $\qquad$ *and* $\qquad$ $\sqrt{2+\sqrt{3}}=\dfrac{\sqrt{3}+1}{\sqrt{2}}$.

18. a) Verify the equality: $\left(4-2\sqrt{3}\right)\left(7+4\sqrt{3}\right)=\left(\sqrt{3}+1\right)^2$.

b) Deduce the solution of the following equation: $\dfrac{x}{4-2\sqrt{3}}=\dfrac{7+4\sqrt{3}}{x}$.

19. Write $A(x)=2\sqrt{9x+27y}+5\sqrt{4x+12y}-4\sqrt{16x+48y}$ in the form of $a\sqrt{b}$

20. Consider a scalene ΔABC such that $AB=\sqrt{7}+\sqrt{6}$, $AC=\dfrac{1}{\sqrt{7}-\sqrt{6}}$ *and* $BC=\sqrt{14}+2\sqrt{3}$.

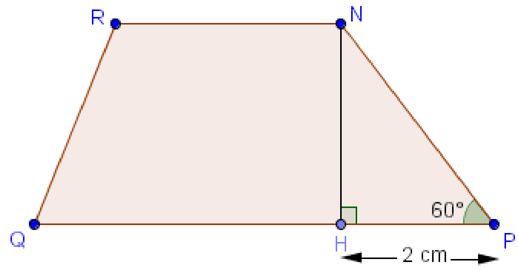
i. Rationalize the denominator of AC.
ii. Prove that ΔABC is a right isosceles triangle.

21. The dimensions of a rectangle are: $r=7+2\sqrt{5}$ *and* $n=3+\sqrt{5}$.

a. Compute the expression: $r-n$.
b. Deduce which dimension r *or* n is the width of the indicated rectangle.
c. Find the measure of the rectangle's diagonal.
d. Calculate the area and the perimeter of the given rectangle.

22. Given the two values of: $X=4+6\sqrt{12}-2\sqrt{27}$ & $Y=2-3\sqrt{48}+4\sqrt{75}$.

a. Simplify the given numerical expressions.
b. If X and Y represent the measures of the bases of a trapezoid $RNPQ$, precise the measure of sides RN and that of PQ.
c. Calculate the measure of the altitude NH.
d. Deduce the area of the given trapezoid.



23. Given in a plane the three points $A, B \& C$ where $AB = 2\sqrt{3}; BC = \sqrt{75}$ *and* $AC = \sqrt{147}$.

a. Verify that $AB + BC = AC$.

b. What can you deduce about these three points?

24. Given a *square* ABCD of side $\left(1+\sqrt{5}\right)cm$.

a. Enclose $\left(1+\sqrt{5}\right)$ between two integers.

b. Place $\left(1+\sqrt{5}\right)$ on a number line.

c. Calculate the area of the given square.

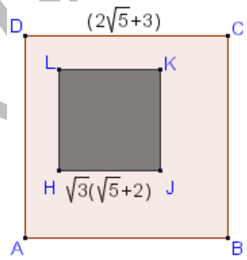
d. Find the radius of a circle (C) circumscribed about the given square.

25. Consider the three points $A, B \& C$ such that $AB = 5\sqrt{3}; BC = \sqrt{192}$ *and* $AC = \sqrt{27}$.

a. Prove that the given points are collinear.

b. Assume that (*P*) is a circle of diameter $[AB]$ and center *O*.

Calculate the measure of the tangent $[CT]$ to (*P*).



26. 1) Expand the following $\left(2\sqrt{5}+3\right)^2$ *and* $\left[\sqrt{3}\left(\sqrt{5}+2\right)\right]^2$.

2) Let $\mathscr{A}$ be the area of the un-shaded domain (part) of the two given squares.

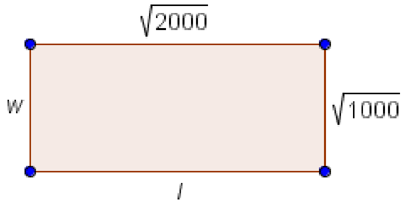
Prove that $\mathscr{A}$ is an integer.

27. Consider the following expressions: $X = \dfrac{7-3\sqrt{5}}{3+\sqrt{5}}$ *and* $Y = \left(2-\sqrt{5}\right)^2$

a. Show that: $X = Y$

b. Prove that: $\sqrt{\dfrac{7-3\sqrt{5}}{3+\sqrt{5}}} + \dfrac{4\sqrt{5}-5}{\sqrt{5}}$ is an integer.

28. Consider the following rectangle:



a. Write the dimensions of the adjacent rectangle in simplest form possible.

b. Judge the relation $l = 2w$.

c. Express the area of the rectangle in the form $a\sqrt{2}$ where *a* is a natural number.

29. Consider the two fixed points *A* and *B* and the variable point *M* to be the same plane.

a. Assume that $MA = \sqrt{x^4 + 2x^2 + 1}$ and $MB = (2x+3)(x+1) - x(x+5) - 2$.

i. Compare the measures of $MA \& MB$.

ii. Deduce in this part the locus of point *M*.

b. Now, take $AB = 5, MA = 2x+1 \& MB = 2\sqrt{6-x^2} - x$.

i. Verify that sum of the squares $MA^2 + MB^2$ is constant.

ii. Deduce again the locus the point *M*.

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