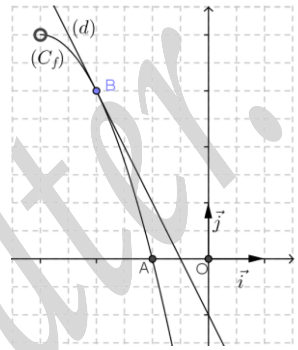
I- If f is a function defined by: $f(x) = ax^3 + bx^2 - 3x^3 + 2x^2 + 5c - 1$, then determine the real numbers $a, b \,\&\, c$ so that $f(0) = f'(2) = 3 \,\&\, f'(-1) = 5$.

II- Consider in the system $(O, \vec{i}, \vec{j})$ the function f defined by: $f(x) = ax^2 + bx + c$ and its representative curve C_f and the tangent (d) to C_f at B.



a. Determine the values of x for which f is defined.

b. Use the graph to find coordinates of the points $A \,\&\, B$.

c. Deduce the numerical values of $a, b \,\&\, c$.

III- Consider the following table of variations of the differentiable functions $f \,\&\, g$.

Values of x	$-\infty$	0	$+\infty$
Sign of $f'(x)$	$-$		$+$
Variation of f	2 ↘	-2	↗ 2

Values of x	0	$+\infty$
Sign of $g'(x)$	$-$	
Variation of g	$+\infty$ ↘	+1

a. Use table to determine the:

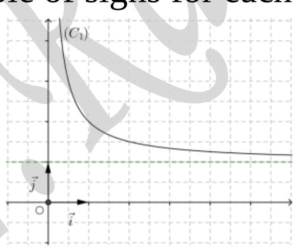
i. Domain and range of $f \,\&\, g$.

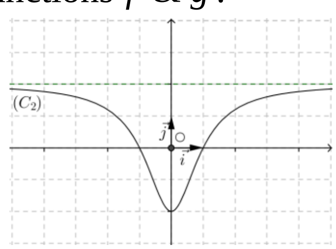
ii. $f'(0)$. What does is tell you?

iii. Asymptotes of the functions $f \,\&\, g$.

b. Relate each table with its appropriate graph.

c. Deduce the table of signs for each of the functions $f \,\&\, g$.





IV- Let f be a function defined by $f(x) = \dfrac{|x|}{|x|+1}$ and (C) its representative curve.

1. Study the differentiability of f at $x_0 = 0$.
2. Write the equations of the semi-tangents drawn from O to (C) (***Al-Ahlia, P:140, Ex:21***)

V- Consider the function f defined by: $f(x) = \dfrac{2x^3 - 3}{(x-1)^2}$

a. Determine the limits of $f(x)$ at the open boundaries of f.

b. Determine the derivative of $f(x)$.

VI- Consider the function f defined by: $f(x)=\begin{cases} \dfrac{rx+1}{x-1} & x<0 \\ nx^2+kx+3 & x\geq 0 \end{cases}$

Determine the real values of r,n & k so that, the representative curve of f has a horizontal tangent at the point $S(2,-1)$ and a horizontal asymptote of equation $y=-1$

VII- Consider the function f defined by: $f(x)=\begin{cases} \dfrac{2a+1}{x-1} & x\leq 0 \\ bx^2+cx-3 & x>0 \end{cases}$

a. Determine the values of a,b & c so that, f is continuous at $x=0$, and its curve, C_f, admits the tangent $(T): y=-2x-1$ at a point of abscissa $x=1$.

b. Let $a=1, b=-2$ & $c=2$

i. Calculate: $\lim\limits_{x\to\pm\infty} f(x)$.

ii. Does C_f admit an asymptote that is parallel to either of the coordinate axis? Justify.

c. Starting from the definition of derivative, study the differentiability of f at $x=0$ and interpret its graphical meaning.

d. Determine the coordinate of the:

i. Point A, at which C_f admits a horizontal tangent.

ii. Points of C_f where the tangent is perpendicular to the straight line $(d): 2x+y=0$

VIII- Consider the function r defined by: $r(x)=ax+b+\dfrac{c}{x-2}$ where a,b & c are real numbers.

a. Find the domain of definition r.

b. Determine the values of a,b & c, so that C_r has a horizontal tangent at $I(1,1)$ and another tangent at a point $B(0,y_B)$ whose slope is 0.75.

c. Assume in this part that $a=b=c=1$

i. Calculate: $\lim\limits_{x\to\pm\infty} r(x)$.

ii. Calculate: $\lim\limits_{x\to\pm\infty}[r(x)-(x+1)]$. Interpret the result graphically.

iii. Study over the domain of r, the relative position of C_r with respect to the straight line (d) of equation $y=x+1$.

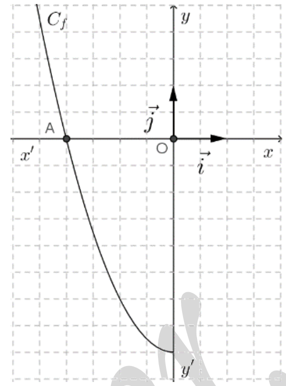
IX- Determine the real values of m so that the curve of a function f defined by

$f(x)=\dfrac{2mx-1}{(m-3)x+5}$ admits at the point of abscissa 2 a tangent of director coefficient -1.

X- Given that $f(3)=1$ & $f'(3)=-3$.

Determine the derivative of $\dfrac{1}{[f(x)]^2}$ at $x=3$.

XI- Let $f\ \&\ g$ be two functions defined over $\mathbb{R}$ by: $f(x)=x^2-4$, $g(x)=|f(x)|$ & part of C_f.



1) Determine the domain of $f\ \&\ g$ and study their parity.
2) Deduce the type of symmetry that curve of f admits.
3) Study the signs of $f(x)\ \&\ g(x)$.
4) Complete the graph of f and deduce that of g.
5) Is g differentiable at $x=2$? Justify.

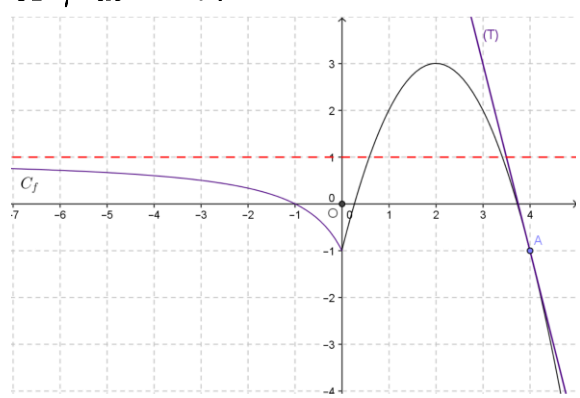
XII- Take in the orthonormal system $(O, \vec{i}, \vec{j})$ the function h to be a function defined by:

$h(x)=\dfrac{x^2+3x+2}{x+5}$ and its representative curve C_h.

a. Find limits of h at the open boundaries of its domain.
b. Deduce the equations of the asymptotes that are parallel to either of the coordinate axis.
c. Prove that the point $I(-5,-7)$ is the center of symmetry of C_h.
d. Determine the derivative of h, and then set up the table of variation.
e. Prove that $h(x)$ can be written in the form: $h(x)=ax+b+\dfrac{c}{x+5}$, where $a, b\ \&\ c$ are real numbers to be determined.
f. Deduce that C_h admits an oblique asymptote whose equation is to be determined.

XIII- Consider the function f defined by its representative curve C_f :

a) Study graphically the continuity and differentiability of f at $x=0$.
b) Determine graphically:



i. $\lim\limits_{x\to-\infty} f(x)$. Interpret your result graphically.
ii. $f'(2)$. Justify
iii. $\lim\limits_{x\to1} f(x)$

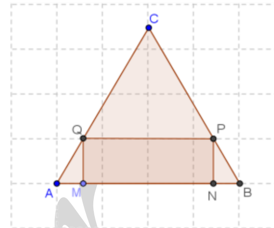
c) Find the equation of (T) the tangent to C_f at A.
d) Deduce the value of $f'(4)$
e) In this part suppose that f is defined by: $f(x)=\begin{cases}\dfrac{ax+b}{x-1} & x\le 0\\ cx^3+dx^2+4x+e & x>0\end{cases}$

i. Utilize the given graph to prove that $a=b=1, c=0\ \&\ d=e=-1$.
ii. Study starting from the definition, the differentiability of f at $x=0$.

f) Find the coordinate of the point(s) where C_f admits a tangent parallel to: $y=2x+1$

XIV- How many maxima does the function f defined by: $f(x)=x^4-2x^2+7$ admit? Justify.

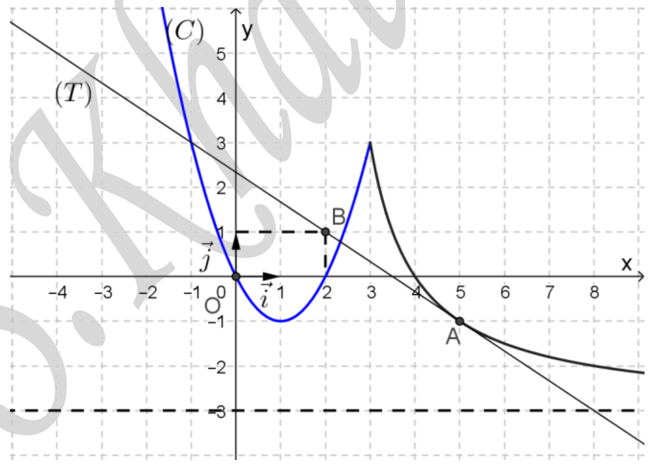
XV- ABC is an equilateral triangle of side $4cm$ .Let $MNPQ$ be a rectangle inscribed in this triangle. Suppose $AM = x\,cm$. (***Al-Ahlia, P:141, Ex:27***)



1. Encircle x between two integers.
2. Determine the measure of side $[NP]$.
3. Calculate the area of the given rectangle.
4. For what values of x is the area of the given triangle *maximum*?

XVI- Consider a function *f* defined by its representative curve (*C*). The tangent (*T*) to (*C*) at the point *A*(5 ; 4) passes through *B*(2 ; 1). (***Test: AlMahdi,2013-2014***)

Use the curve (C) to answer the following questions:



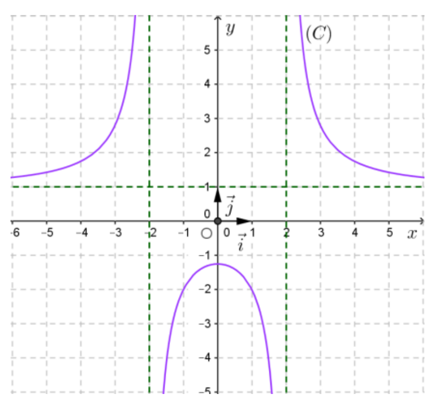
1) Give the domain of definition of *f*.
2) Determine $\lim\limits_{x\to+\infty} f(x)$; $\lim\limits_{x\to-\infty} f(x)$; $\lim\limits_{x\to 3} f(x)$.
3) Study graphically the continuity & the differentiability of *f* at point *x = 3*.
4) Determine $f'(1)$ *and* $f'(5)$.
5) Given the function g defined by: $g(x)=\dfrac{1}{f(x)-3}$

    a) Give the domain of definition of *g*.

    b) Determine *g*(5) & *g'*(5) .

6) Suppose that $f(x)=\begin{cases} x^2-2x & if\ x\le 3 \\ \dfrac{-3x+12}{x-2} & if\ x>3 \end{cases}$.
7) By using the definition of the derivative, study the differentiability of *f* at *x* = 3.

XVII- Consider the curve (C) of a function f .



a. Compare with justification:
   $f(-4)........f(-3)$; $f(100)......1$;
   $f'(1)......f'(-3)$ and $f'(2.3).....f'(1300)$

b. The tangent to (C) at a point of abscissa $x=1$, is of equation: $2x+y=0$

c. Calculate: $f(1)$ & $f'(1)$

d. Let g be a function defined by: $g(x)=\dfrac{x^2}{2-f(x)}$.

e. Prove that g is defined for all real values of x.

f. Prove that equation of tangent to C_g at a point of abscissa $x=1$ is: $y+\dfrac{1}{4}=\dfrac{3}{2}(x-1)$.