Lycée Des Arts | Mathematics | 9th-Grade

Name: . . . . | Polynomials & Fractional Expressions | W.S-4

1) Decompose the following into products of two or more factors and then solve each one if possible:

$A=(3a+1)(a+1)+a^2-1$

$B=(6x^2-12x+6)+(3x^2-3)-(x-1)(2x+1)$

$C=x^5+x^2-x^3-1$

$D=(a-2)(4a^2+4a+1)-(a-2)^3$

$E=4x(3x-1)-(x+2)(3x-1)-3x+1$

$F=a^2+b^2-x^2-y^2-2ab-2xy$

$G=4y^2-9+(2y+3)(y-5)$

$H=(x-3)(2x+7)+(2x-6)(3x-1)-(9-3x)(x+1)$

$I=(x+7)(3x+4)+(9x^2+24x+16)$

$J=3x^2-12+(x-4)(2-x)-(x^2-4x+4)$

$K=a^3+a^2-4a-4$

$L=4x^2-4x+1-(1-2x)(3x+5)-12x^2+3$

$M=xy-3x-2y+6$

$N=6(x^2-16)-(3x+1)(x-4)+(8-2x)(x+2)$

$O=10ab-2+4a-5b$

$P=25(3x-y)^2-16(5x+3y)^2$

$Q=(2x-3)(x-1)^2-4(2x-3)$

$R=(4x-3)(-x+5)+(x-1)(x-5)+(2x-5)(-x+5)$

$S=x^6-x^4-x^2+1$

$T=4(x^2+14x+49)-2x^2+98$

$U=\left(\frac{x}{4}-\frac{1}{3}\right)^2-\left(\frac{5x}{4}-\frac{2}{3}\right)^2$

$V=(3x+2)^2+2(3x+2)(x-1)+(x-1)^2$

$W=\frac{r^2}{4}-\frac{rn}{2}+\frac{n^2}{4}$

$X=(2a-3)^2-2(2a-3)+1$

$Y=x^2-5x+4$

$Z=x^2-x-12.$

2) Consider the algebraic expressions:

$$E=2x(3x+2) \text{ and } F=9x^2+12x+4.$$

a. Solve the equation $E=0$.

b. Calculate for $x=\sqrt{2}$; the numerical value of F. Give the result in the form of $a+b\sqrt{2}$ where a and b are two integers.

c. $i)$ Factorize the expression $E-F$.
   $ii)$ Deduce the values of x for which $E=F$.

3) Given the polynomial $P(x)=(3x-m)(4x+7)-9x^2+25$.

a. Does the degree of the given polynomial depend on m? Explain.

b. Calculate m so that (-2) is a root of $P(x)$.

c. Factorize $P(x)$, then deduce its roots so that $m=5$.

4) Given the two polynomials: $P(x)=x^2+2x-3$ and $Q(x)=(x+1)^2-a$.

a. Calculate a so that the polynomials are identical.

b. Deduce the roots of $P(x)$.

c. Verify that $P(\sqrt{5}-1)$ is an integer.

5) Consider the polynomial: $S(x)=(2m-1)x^3+(3m-4)x^2+5x-2$.

1- Determine m, if 1 is a zero of $S(x)$.

2- For which values of m, is $S(x)$ a 2^{nd} degree trinomial?

3- Indicate the degree of $S(x)$, if $m\neq 1/2$?

6) Consider the polynomial $N(x)=x^2-6x+5$.

1- Show that $N(x)+4$ is a perfect square, deduce the factorization of $N(x)$.

2- Calculate the dimensions of a rectangle whose area is $5m^2$ and its perimeter is $12m$.

3- Solve $N(x)>(x-3)^2$. Interpret your result.

4- Calculate $a, b, and\ c$ if $N(x)$ is identical to $(a-b)x^2-2(a+b)x+4a+c$.

7) The unit of length is cm and x designates a non-zero positive integer. Consider a rectangle of dimensions $(x+1)$ and 4, and an equilateral triangle of side $(x+1)$. Designate by P_1 the perimeter of the rectangle and by P_2 that of the equilateral triangle.

a- Express P_1 and P_2 in terms of x.

b- For what values of x we have $P_1=P_2$?

c- Find x so that: $P_1<P_2$. Interpret obtained result.

d- Deduce for what values of x: $P_1>P_2$.

8) Given the expressions: $P(x)=(5x-2)(5x+8)$ *and* $Q(x)=(5x+3)^2-25$.

1- Expand then reduce $P(x)$.

2- Factorize $Q(x)$.

3- ABC is a right angled triangle at A such that: $AB=5$ and $BC=5x+3$, where $x\geq 0$.

i. Show that: $AC^2=25x^2+30x-16$.

ii. For $x=2$, calculate the perimeter and the area of the triangle ABC.

9) Given the polynomial: $F(x)=(a+1)x^3-(b-1)x^2+cx+1$ where x is the variable.

1- Determine $a, b, and\ c$ so that $G(x)=F(x)-1$ is identical to zero.

2- Determine $a, b, and\ c$ so that $F(x)$ is identical to $(x-1)(x+1)(2x-1)$.

10) Let $Q(x)=(2x-1)(x-1)^2-4(2x-1)$.

i. Factorize $Q(x)$.

ii. Solve the equation $Q(x)=0$.

2- Let $H(x)=\dfrac{Q(x)}{(x-1)(x+1)(2x-1)}$.

i. For what values of x the fraction $H(x)$ is defined?

ii. Simplify $H(x)$.

iii. Solve $H(x)=0$ and $H(x)=2$.

11) Consider the following equation: $3mx-2=2x+5$.

a. Solve for x.

b. For what values of m is x defined?

c. Evaluate x *for* $m=-1$ & $m=\sqrt{2}$.

12) If $n=\frac{-1+\sqrt{5}}{2}$, then compare n^2 *and* $1-n$.

Explain without any calculations, why n is a solution of the equation: $x^2+x-1=0$.

13) Consider the polynomial: $R(x)=(x-2)^2+5(x-3)(2-x)+x^2-4$.

1- Develop and reduce $R(x)$.

2- Show that $R(x)=-3x^2+3(7x-10)$.

3- Write $R(x)$ as a product of two or more factors of first degree order.

4- Deduce the roots of $R(x)$.

5- Calculate $R(\sqrt{2})$.

14) Consider the polynomial: $N(x)=4-x^2+(x-2)(2x+3)$.

a) Factorize $N(x)$ and then deduce its roots.

b) Develop and reduce $N(x)$ and then show that $N(x)+2=x(x-1)$.

c) Calculate $N(2\sqrt{2})$ & $N(\sqrt{3}-1)$.

15) Given the polynomial $P(x)=x^2-m+2(x-1)(x-2)$.

1) Determine the value of m so that $+2$ is a root of $P(x)$.

2) Factorize $P(x)$, if $m=4$.

3) Solve $P(x)=0$.

4) Give all natural numbers that verify $P(x)\geq 3x^2-9$.

16) Consider the following polynomials:

$P(x)=(2x+1)^2-(3x-5)^2$ *and* $Q(x)=4x^2-25-(3x+1)(-2x+5)-20x+50$.

a- Develop, reduce then order $P(x)$ & $Q(x)$.

Factorize $P(x)$ & $Q(x)$.

b- Solve the following equations:

i- $P(x)=0$.

ii- $P(x)=Q(x)$.

iii- $Q(x)=10x(x-3)$.

17) Let $R=4x^2-4x-8$ *and* $N=2x(x-2)^2$.

1- Show that $R+9$ is a perfect square, then factorize R.

2- Simplify $K=\frac{R}{N}$, then solve $K=0$.

3- Let $F=(2x-a)^2-ab$, calculate a *and* b when $R\equiv F$.

18) The measures of sides of a ΔABC are: $AB=3x+6$; $AC=4x+8$ & $BC=5x+10$.

1- Show that ΔABC is right at A. (Where $x\geq 0$.)

2- Let $[AH]$ be the height relative to $[BC]$. Show that $\overline{AH}=\frac{12}{5}(x+2)$.

3- Compute the area S of the ΔABC, for $x=\sqrt{2}$, and express the result in the form $a+b\sqrt{2}$, where a & b are two integers to be determined.

19) Consider the polynomial: $P(x)=(x+8)(x^2+6x+10)-(x+8)(x+6)$.

1. Develop, reduce and order $P(x)$.
2. Factorize $P(x)$, then solve the equation $P(x)=0$.
3. Consider the fraction: $F(x)=\dfrac{P(x)}{x^2+9x+8}$.

a) Write the domain of definition of $F(x)$.

b) Simplify $F(x)$, then solve the equation $F(x)=10$.

20) Consider the trinomials:

$R(x)=(a+b)x^2+(a-b)x+5c+1$ *and* $N(x)=(2a-1)x^2-(2-3b)x+3c-1$.

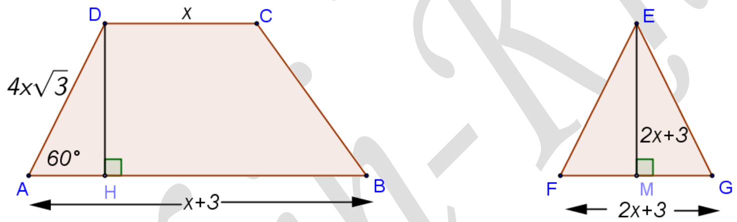
Find the numerical values of $a, b,$ *and* c so that $R(x)\equiv N(x)$.

21) Given $P(x)=(2x+3)^2+(6x+9)(x-2)-4x^2+9$ *and* $Q(x)=4x^2+12x+9$.

1- Factorize $P(x)$ *and* $Q(x)$.

2- Suppose in this part $P(x)=3x(2x+3)$ *and* $Q(x)=(2x+3)^2$. Solve $P(x)=Q(x)$.

3- In this part, x represents a unit of length, such that $x>1$.



i. Calculate HD, then prove that the area of the trapezoid is: $P(x)$.

ii. Compute in terms of x the area of the triangle EFG.

iii. Find x so that the area of the trapezoid is double than that of the triangle EFG.

22) MAN is a right triangle at A, such that $AM=2x+1$ & $AN=x+3$.

1) Let $P(x)$ be a polynomial defined by $P(x)=MN^2$.

a. Find the expression of $P(x)$.

b. Compute x such that ΔMAN is isosceles, then work out MN^2.

c. Solve $P(x)=50$.

d. Assume that $S(x)$, is area of ΔMAN, write expression of $S(x)$, then calculate $S(2)$.

2) In this part $x>1$, and I is a point on $[AN]$ such that $\overline{AI}=2x-1$.

i. Find the area of ΔINM .

ii. Can you find a value of x, so that Δ's INM & IAM have the same area? Explain.

23) If $N(x)=(x-2)^2-(x-1)(x-4)$, then Complete the following table:

x	$x-2$	$(x-2)^2$	$x-1$	$x-4$	$(x-1)(x-4)$	$N(x)$
10						
100						

1- Deduce the developed form of $N(x)$.

2- Deduce the value of $E=1234^2-1235\times1232$ without using a calculator

24) In this exercise, x is a unit of length expressed in cm.
$ABCD$ is a rectangle such that $AB = x - 2$ & $BC = 3x + 1$.
MNP is a triangle right at M, such that $MN = 2(x-2)$ & $MP = x + 10$.
Let S be the area of $ABCD$ and S' be the area of MNP.

1) Calculate in terms of x the areas S and S'.
2) Factorize $S - S'$.
3) Calculate x, so that $S = S'$.

25) *Right triangles:* Given that x is a strictly positive number.

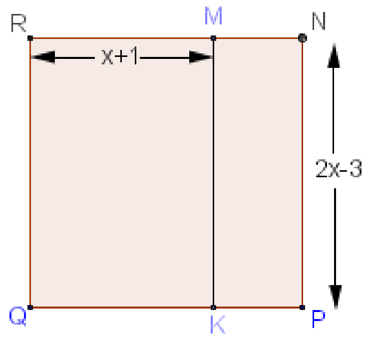
1. For what values of x is a triangle of sides $x-1, x, x+1$ right?
2. Find the measure of sides of this triangle.
3. If $3n, 4n \& 5n$, are the dimensions of a triangle, where n is non-zero natural number:

a) Complete the following table:

Value of n	$a = 3n$	$b = 4n$	$c = 5n$	a^2	b^2	c^2	$a^2 + b^2$
2							
3							

b) Which entries of the above table are equivalent? Deduce the nature of this triangle.
c) State your conclusion?

26) Consider the adjacent figure, where quadrilateral $RNPQ$ is a square:



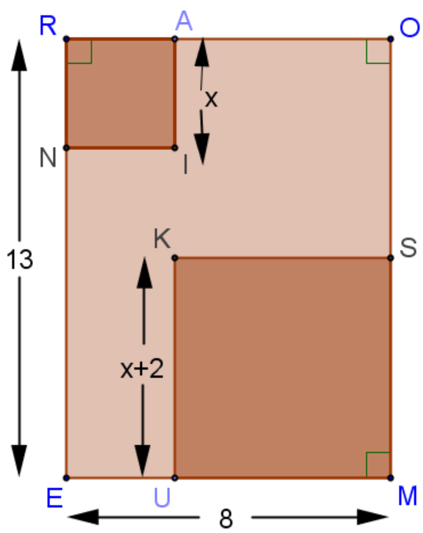
1- Solve the inequality: $2x - 3 \geq x + 1$.
Note: in what follows $x \geq 4$.

2- Now, let $A(x)$ represent the area of quadrilateral $MNPK$.

i. Show that $A(x) = (2x-3)^2 - (2x-3)(x+1)$.
ii. Expand $A(x)$.
iii. Factorize $A(x)$.
iv. Solve $(2x-3)(x-4) = 0$.
v. For what value of x, does the area of $MNPK$ equal to zero?

27) ***Part-A:*** Let $f(x) = (x+2)^2 - 4x^2$.

i. Express $f(x)$, as a product of two binomials.
ii. Find the roots of $f(x)$.



Part-B: Given that x is an integer in cm and $0 < x < 6$.
Consider the rectangle $ROME$ and the two squares $RAIN$ and $MUKS$.

1- Show that $A(x)$, the area of the un shaded region is given by: $A(x) = 104 - x^2 - (x+2)^2$.

2- a) Expand then reduce $A(x)$.
b) Verify that: $A(x) - 52 = -2(x-4)(x+6)$.
c) For what value of x the area of the un shaded region is $52cm^2$.

3- Use ***Part-A*** to find the value of x so that the area of $MUKS$ is four times that of $RAIN$.

28) Consider the polynomial: $E(x) = x^3 + 3 - 3x^2 - x$.

1- Write $E(x)$ in the form of a product of three first degree <u>binomials</u>.

2- a) Show that: $E(x+1) = x(x^2 - 4)$, then factorize: $E(x+1) - E(x-1)$.

b) Find all non-zero integers of x, so that $E(x+1) = E(x-1)$.

29) If $x - \frac{1}{x} = 2$, then calculate $x^2 + \frac{1}{x^2}$.

30) Consider a rectangle of perimeter $20m$ and of length ℓ.

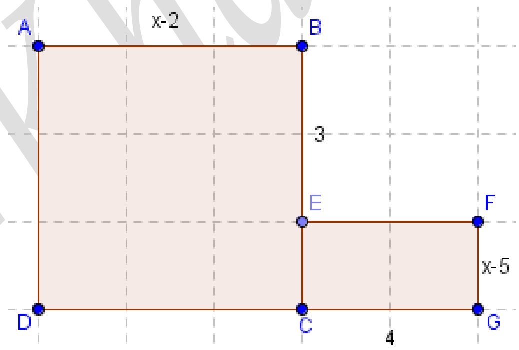
a) Express the area A in terms of ℓ.

b) Show that $A = 25 - (\ell - 5)^2$.

c) Deduce the value of ℓ for which the area of the given rectangle is maximum.

31) Consider the two expressions: $f(x) = 4(x-5) + (x-2)^2$ & $q(x) = (x-2)^2 - 12(x-5)$.

1. Expand and reduce $f(x) \& q(x)$.
2. Factorize $f(x) \& q(x)$.
3. Solve:
   i. $f(x) = 20$.
   ii. $q(x) = 0$.
   iii. $q(x) = 25$.



4. In what follows x is a measure expressed in *cm* such that $x > 5$. In the above figure designate by $S(x)$ and $S'(x)$ the respective areas of $ABCD$ and $EFGC$.
   a. Express $S(x)$ and $S'(x)$ in terms of x.
   b. Determine area of $EFGC$ if : $S(x) = 3S'(x)$.

32) One of the greatest mathematicians of his time, Leonhard Euler, claimed that the number

$P = n^2 + n + 41$ is always a prime number for $n = 0, 1, 2, 3, 4, 5, \ldots$

a) Find the value of P for $n = 0, 1, 2, 3, 4, 5, 6, 7$ *and* 8.

b) By making a suitable substitution, show that Euler's claim was worng.

33) If $x = 2\sqrt{2} + 1$, then

1- Calculate the numerical values of: $x^2 \& 2x + 7$.

2- Compare the obtained answers.

3- Deduce that: $x - 2 = \frac{7}{x}$.

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