1. Decompose the following into ***products*** of two or more factors and then ***solve*** each one:

$A = a^3 + a^2 - 4a - 4$

$B = 4\left(x^2 + 14x + 49\right) - 2x^2 + 98$

$C = x^5 + x^2 - x^3 - 1$

$D = (a-2)\left(4a^2 + 4a + 1\right) - (a-2)^3$

$E = 4x(3x-1) - (x+2)(3x-1) + 3x - 1$

$F = (2x+1)(4x+3) - 5x(4x+3) + (x-1)(4x+3)$

$G = (2x-3)(x-1)^2 - 4(2x-3)$

$H = (x-3)(2x+7) + (2x-6)(3x-1) - (9-3x)(x+1)$

$I = (x+7)(x+4) + \left(9x^2 + 24x + 16\right)$

$J = 3x^2 - 12 + (x-4)(2-x) - \left(x^2 - 4x + 4\right)$

$K = x^6 - x^4 - x^2 + 1$

$L = (4x-3)(-x+5) + (x-1)(x-5) + (2x-5)(-x+5)$

$M = 2x\left(x^2 - 1\right) - x(x+1)$

$N = 6\left(x^2 - 16\right) - (3x+1)(x-4) + (8-2x)(x+2)$

2. Choose, with ***justification*** the only correct answer:

No.	Problem	Expected answer		
		A	B	C
1.	The equation: $x(x+1) = (x-3)^2$ is verified for	one value of x	all values of x	no value of x (impossible)
2.	If e is any non-zero real number and $(e-1)$ is a solution for $p(x) = x^2 + 4x - m + 4$, then $m =$	$(e-1)(e+1)$	$(e-1)^2$	$(e+1)^2$
3.	The equation $\frac{4x^2 - 9}{2x+3} = 0$ admits	A unique solution $x = \frac{-3}{2}$	Two solutions $x = \frac{-3}{2}$ and $x = \frac{3}{2}$	A unique solution $x = \frac{3}{2}$
4.	If $\left(\frac{1}{2}\right)$ is a root of $p(x) = (3x-a)(2x+a)$, then	$a = \frac{3}{2}$ *or* $a = -1$	$a = 6$ *or* $a = \frac{-1}{4}$	$a = \frac{-3}{2}$ *or* $a = 1$

3. Consider the following equation: $\frac{x-1}{x} = \frac{x}{x-1}$.
   - ***a.*** For what values of x is the above expression valid?
   - ***b.*** Does the above equation admit a solution in set of natural numbers? Explain.

4. Consider the following equation: $3mx - 2 = 2x + 5$.
   - ***a.*** Solve for x.
   - ***b.*** For what values of m is x defined?
   - ***c.*** Evaluate x *for* $m = -1$

5. Given the polynomial $P(x)=(3x-m)(4x+7)-9x^2+25$.
   - ***a.*** Calculate m so that (-2) is a root of $P(x)$.
   - ***b.*** Factorize $P(x)$, then deduce its roots so that $m=5$.

6. Consider the polynomial $E(x)=(2x-3)^2+(2x-3)(5x+1)$
   - ***a.*** Develop $E(x)$ and reduce in ascending order.
   - ***b.*** Factorize $E(x)$.
   - ***c.*** Solve $E(x)=0$.

7. Consider the algebraic expressions: $E=2x(3x+2)$ *and* $F=9x^2+12x+4$.
   - ***a.*** Solve the equation $E=0$.
   - ***b.*** *i*) Factorize the expression $E - F$.
     ii) Deduce the values of x for which $E=F$.

8. The unit of length is *cm* and x designates a positive number. Consider a rectangle of dimensions $(x+1)$ and 4, and an equilateral triangle of side $(x+1)$. Designate by P_1 the perimeter of the rectangle and by P_2 that of the equilateral triangle.
   - ***a.*** Express P_1 and P_2 interms of x.
   - ***b.*** For what values of x we have:
     - ***i.*** $P_1=P_2$?
     - ***ii.*** $P_1< P_2$?

9. Given the expressions: $P(x)=(5x-2)(5x+8)$ *and* $Q(x)=(5x+3)^2-25$.
   - ***a.*** Expand and reduce $P(x)$.
   - ***b.*** Factorize $Q(x)$.
   - ***c.*** *ABC* is a right angled triangle at *A* such that: $AB=5$ and $BC=5x+3$. Where x is a positive number.
     - ***i.*** Show that: $AC^2=25x^2+30x-16$.
     - ***ii.*** For $x=2$, calculate the perimeter and the area of the triangle *ABC*.

10. Let $Q(x)=(2x-1)(x-1)^2-4(2x-1)$.
    - ***a.*** Solve the equation $Q(x)=0$.
    - ***b.*** Let $H(x)=\dfrac{Q(x)}{(x-1)(x+1)(2x-1)}$.
      - ***i.*** For what values of x the fraction $H(x)$ is defined?
      - ***ii.*** Simplify $H(x)$.
      - ***iii.*** Solve $H(x)=0$
      - ***iv.*** Does the rational expression $H(x)=2$ admit a solution?

11. Consider the polynomial: $R(x)=(x-2)^2+5(x-3)(2-x)+x^2-4$.

a. Express $R(x)$ in the form, ax^2+bx+c, where $a,b\,\&\,c$ are integers to be determined.

b. Show that $R(x)+3x^2=3(7x-10)$.

c. Write $R(x)$ as a product of two or more factors of first degree order.

d. Deduce the roots of $R(x)$.

12. Consider the polynomial: $N(x)=4-x^2+(x-2)(2x+3)$.

a. Factorize $N(x)$ and then deduce its roots.

b. Develop and reduce $N(x)$ and then show that $N(x)+2=x(x-1)$.

13. Given the polynomial $P(x)=x^2-m+2(x-1)(x-2)$.

a. Determine the value of m so that $+2$ is a root of $P(x)$.

b. Factorize $P(x)$, if $m=4$.

c. Solve $P(x)=0$.

d. Give all natural numbers that verify $P(x)\geq 3x^2-9$.

14. Let $F(x)=\dfrac{x^2-9+5(2x-6)+3-x}{x^2-6x+9}$.

a. Indicate for which values of x is $F(x)$ not defined?

b. Deduce the domain of definition of $F(x)$.

c. Simplify $F(x)$, then find roots of $F(x)$.

15. 1) Expand and reduce the expression $E=(x-1)^2-(x-2)(x-3)$.

2) Use the preceding result to calculate the expression $A=(9999)^2-9998\times 9997$.

16. Consider the following polynomials:

$$P(x)=(2x+1)^2-(3x-5)^2 \quad \textit{and} \quad Q(x)=4x^2-25-(3x+1)(-2x+5)-20x+50.$$

a. Develop, reduce then order $P(x)\,\&\,Q(x)$.

b. Factorize $P(x)\,\&\,Q(x)$.

c. Solve the following equations:

i. $P(x)=0$.

ii. $P(x)=20$.

iii. $P(x)=Q(x)$.

17. Consider the polynomials:

$R(x)=(2x-3)^2-x(8-5x)-4x+7.$ & $N(x)=9x^2-16+(8-6x)(x+1)$

a. Prove that: $P(x)=(3x-4)^2$.

b. Factorize $N(x)$.

c. Expand, reduce and order $N(x)$.

d. Solve the equations:

 i. $P(x)=25$.

 ii. $N(x)=-8$.

e. Compute the roots of $N(x)$.

f. Let $K(x)=\dfrac{P(x)}{N(x)}$.

 i. Simplify $K(x)$.

 ii. Solve $K(x)=-3$.

18. Given: $A=49-(2x-3)^2$ & $B=(x-5)(x+2)+(5-x)(3-x)$.

a. Factorize then reduce A & B.

b. Let $C=\dfrac{A}{B}$.

 i. Simplify C.

 ii. Solve the equations: $C=1$ & $C=-1$.

 iii. Evaluate the roots of C.

19. Consider the two polynomials: $f(x)=4x^2-1+(2x-1)^2-(2-4x)(x+2)$;

$g(x)=(3x+1)^2-(x+2)^2$

1) Develop and reduce $f(x)$.

2) Show that $f(x) = 2(2x-1)(3x+2)$.

3) Write $g(x)$ in the form of a product of two factors of the first degree.

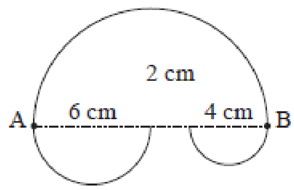
4) If $h(x)=\dfrac{f(x)}{g(x)}$, then simplify $h(x)$, and solve the equation $h(x)=2$.

20. Consider the algebraic expression: $R=n^3-n$, where n is a natural number
Prove that R is:

a. A product of three consecutive numbers.

b. Divisible by 6 for all natural values of n.

21. Which is the shortest path from A to B given that each curve is a semi-circle?



Mastering problems		
Chapter	*Exercises*	*Pages*
CH-: Powers	1,3,7,9,10,12,13,17 $\to$ 25	93 $\to$ 99