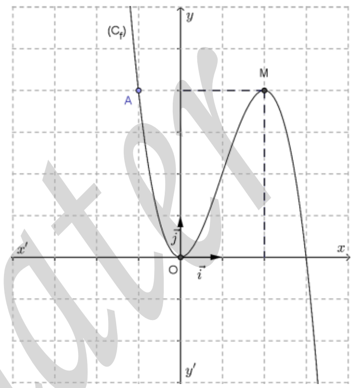
Al- Mahdi High	Mathematics	11^{th}-Grade
Name:	Polynomial functions	W.S-5

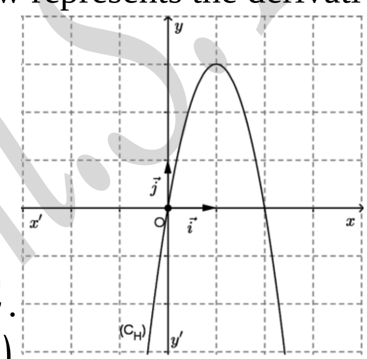
I- Consider the representative curve (C_f) of a function f, where $(C_f)\cap x-axis=\{0,3\}$, and (C_f) admits a minimum and a maximum at $x = 0$ & at $x = 2$ respectively.

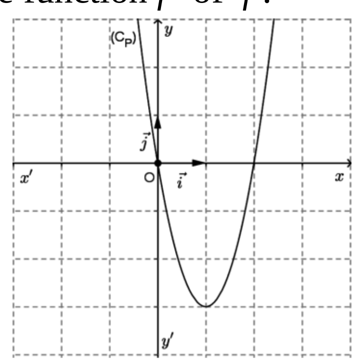
Part-A:



1. Determine the domain of definition of f.
2. Use the graph to:
   i. Solve: $f(x)\leq 0$ & $f(x)>0$.
   ii. Verify that 0 & 2 are roots of $f'(x)$.
   iii. Set up the table of variations of f.
   iv. Show that there exist a real value $\alpha\in]2;3.5[$ so that $f(\alpha)=0$.
3. Discuss graphically according to the real values m the number and the sign of the roots of the equation $f(x)=m$.

Part-B: One of the curves $(C_H)\, or\, (C_P)$ below represents the derivative function f' of f.





a. Determine the curve of f'. Justify your answer.

b. Find graphically $f'(1)$, then deduce the equation of the tangent to (C_f) at $x=1$.

c. Assume that f is defined by: $f(x)=ax^3+bx^2+cx$ .Use (C_f) to find $a,b\,\&\,c$.

d. Determine B, the inflection point of (C_f)

II- Prove that the function h defined by $h(x)=x^3-3x+1$ admits an inflection point, whose coordinates are to be determined.

III- Consider the function f defined over $\mathbb{R}$ by $f(x)=\dfrac{1}{4}x^4-\dfrac{3}{2}x^2+2x+2$ and let (C_f) be its representative curve.

1. Determine the limits of f at $\pm\infty$.
2. Verify that $f'(x)=(x+2)(x-1)^2$, and then draw a table of variation of f.
3. Deduce that $f(x)=0$ admits two distinct roots $\alpha\,\&\,\beta$.
4. Prove that $\alpha\in]-3;-2.5[$ & $\beta\in]-1;-0.5[$;
5. Prove that (C_f) admits two inflection points, then draw (C_f).
6. Deduce (C_g) the representative curve of the function g defined over $\mathbb{R}$ where $g(x)=|f(x)|$.

IV- Consider the function f defined over $\mathbb{R}$ by:

$f(x) = mx^2 - (2m-1)x + m$, where m is a non-zero real parameter and designate by (C_m) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

✓ ***Part-A:***

1. Show that for all m different from 0, (C_m) passes through a fixed point A, whose coordinates are to be determined.
2. Show that $(l): x = \dfrac{2m-1}{2m}$ is the axis of symmetry of (C_m).
3. Let B be the symmetric of A with respect to (l).
   a) Prove that the coordinates of B are $\left(\dfrac{m-1}{m}, 1\right)$
   b) Show that the tangents at A and B to (C_m) are perpendicular.

✓ ***Part-B:*** In this part take $m = -1$

1) Study the variations of f.
2) Write the equations of the tangents to (C_{-1}) from the point $R(2,2)$.

V- Let f be a function defined over $\mathbb{R}$ by $f(x) = x^3 + x^2 + x + 2$ and designate by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$. (***Mastering, P:216, Ex:4***)

1. Show that the point I of (C) of abscissa $\dfrac{-1}{3}$ is a center of symmetry of (C).
2. Study the variations of (C).
3. Does (C) admit an inflection point? Justify.
4. Deduce that the equation $g(x) = 0$ admits a unique root $\alpha \in]-2,-1[$. Then trace (C).

VI- Consider the function f defined by $f(x) = x^2 - (m-2)x + m - 3$, where m is a real parameter and (C_m) its representative curve.

1. Specify the condition for which a curve admits an axis of symmetry.
2. Calculate the numerical value of m, so that:
   a. $x = 1$ is an axis of symmetry of (C_m).
   b. f admits a minimum equals -4.
   c. (C_m) is tangent to the abscissa axis.
3. In this part take $m = 4$

Solve in $\mathbb{R}$

a. $|f(x)| = 1$
b. $f(x) < 4$

VII- Consider the table of variations of a function f whose representative curve, in an orthonormal system, is (C).

x	$-\infty$		-3		-1		0		2		3		$+\infty$
$f'(x)$		+	0	-	‖	+	|	-	0	+	‖	-	
$f(x)$	2		5		‖		-3		-5		‖		-2

1) Copy and complete the above table of variations, then find the domain of definition of f.
2) Find the limits of f at the bounds of the domain of definition.
3) Deduce the equations of the asymptotes of (C). Is f odd? Justify.
4) What is the number of solutions of the equation: $f(x) = 0$? Justify, and then draw (C).

VIII- Let (C_f) be the representative curve of a function f defined by its table of variations:

Part-A:

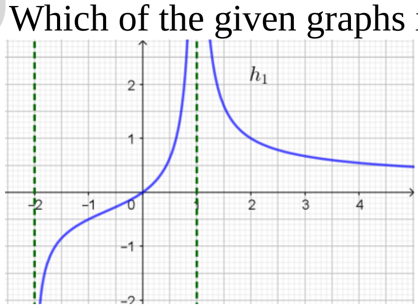
x	$-\infty$		-1		1		$+\infty$
$f'(x)$		+	0	$-$	0	+	
$f(x)$	$-\infty$	↗	4	↘	0	↗	$+\infty$

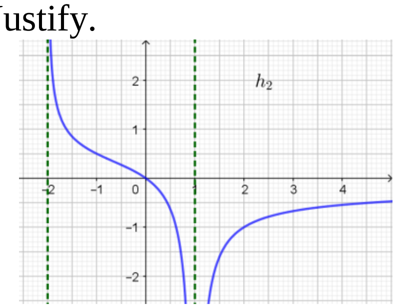
1) Prove that f is not an odd function over its domain.
2) If $f(-2)=0$, then deduce the sign of $f(x)$.
3) Study according to the real parameter m, the number of solutions of $f(x)=m$.
4) Compare with justification, $f(10^6)$ & $f(10^8)$, then trace (C_f).

Part-B:

In this part we admit that $f(x)=x^3-3x+2$ is the image of the given function f.

1. Determine the equation of (L), the tangent to (C_f) at $x=2$.
2. Let g be a real valued function defined by its image: $g(x)=|x^3|-3|x|+2$
   a. Use (C_f) to trace the (C_g).
   b. Is g differentiable at $x=0$? Justify.
3. Let h be another real valued function defined by its image: $h(x)=\dfrac{2x-1}{\sqrt{f(x)}}$
   a. Determine the domain of definition of h.
   b. Calculate the numerical value of $h'(-1)$
   c. Which of the given graphs is that of h? Justify.





IX- Let f be the function defined over $\mathbb{R}$ by $f(x)=x^2-4x+3$ and designate by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$. (***Mastering, P:216, Ex:3***)

1. Study the variations of f and trace (C).
2. Prove that (C) admits an axis of symmetry, whose equation is to be determined.
3. Show ***again*** that the straight line $(\delta): x=2$ is an axis of symmetry of (C).
4. Write an equation of the tangent (T) to (C) at a point A of (C) of abscissa 1.
5. Let (d) be a straight line passing through $B(2,-3)$ and of slope m.
    a. Express in terms of m the equation of (d).
    b. Discuss according to the values of m, the number of points of intersection of (d) & (C).
    c. Deduce the equations of the tangents through the point $B(2,-3)$ to (C).
6. Construct the representative curve (C_g), of the function g defined on $\mathbb{R}$ by $g(x)=x^2-4|x|+3$.