

**I-** Answer with *justification* by *True* or *False*.

1. If  $\vec{AC} = \vec{BD}$  then,  $\vec{AB} = \vec{CD}$ .

2. If  $AC = BD$  then,  $\vec{AC} = \vec{BD}$ .

3. If  $ABCD$  is a rectangle such that  $AB = 6\text{cm}$  &  $AD = 4\text{cm}$  then,  $\|\vec{CB} + \vec{CD}\| = 2\sqrt{13}\text{cm}$ .

4. If  $\vec{AB} = k\vec{CD}$  where  $k \in \mathbb{Z}^*$ , then:

a. Vectors  $\vec{AB}$  and  $\vec{CD}$  are of the same sense.

b. Vectors  $\vec{AB}$  and  $\vec{CD}$  have same direction.

c.  $\|\vec{AB}\| = k\|\vec{CD}\|$ .

5. If  $A, B$  &  $C$  are any three non-collinear point, so that  $\vec{n} = 3\vec{AB} - 2\vec{AC}$  and  $\vec{s} = 3\vec{AB} + 2\vec{BC}$  then the coordinates of  $\vec{n}$  &  $\vec{s}$  in the system  $\left( A; \vec{AB}, \vec{AC} \right)$  are  $\vec{s}(2,1)$  &  $\vec{n}(-3,2)$

6. If  $ABC$  is a right isosceles triangle at  $A$ , so that  $AB = 6\text{cm}$ , and the point  $J$  is defined by  $\vec{JB} = \frac{3}{2}\vec{AC}$ , then the coordinates of  $J$  in the system  $\left( O; \frac{1}{3}\vec{AB}, \frac{1}{3}\vec{AC} \right)$  are  $J(3,5)$

7. In the system  $(O, \vec{i}, \vec{j})$ :

a. The two vectors:  $\vec{u} = \vec{i} + \vec{j}$ ,  $\vec{v} = \vec{i} - \vec{j}$  represent basis.

b. If the vectors  $\vec{a}(3, 2m-1)$  &  $\vec{s} = \vec{i} - 2m\vec{j}$  are collinear, then  $m = 2$

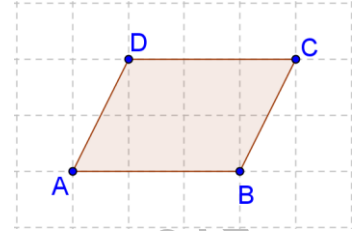
**II-** Consider in a given plane the two distinct points  $A$  &  $B$  and a point  $G$  defined by the vector relation:  $\vec{GB} - 2\vec{AG} = \vec{0}$ .

a. Construct the point  $G$ .

b. If  $N$  is any point of the given plane, then express:  $2\vec{NA} + \vec{NB}$  as a function of  $\vec{NG}$ .

c. Determine the locus of the set of points  $N$  in the plane such that:  $\|2\vec{NA} + \vec{NB}\| = 3\|\vec{NA}\|$

**III-** The adjacent figure represents a parallelogram  $ABCD$ .



- a. Construct the points  $E$  &  $F$  such that  $\overrightarrow{AB} = 3\overrightarrow{BE}$  &  $\overrightarrow{DF} = 3\overrightarrow{AD}$ .
- b. 1) Calculate  $a$  &  $b$  so that,  $\overrightarrow{CF} = a\overrightarrow{AB} + b\overrightarrow{AD}$ .  
2) Calculate  $m$  &  $n$  so that,  $\overrightarrow{EC} = m\overrightarrow{AB} + n\overrightarrow{AD}$ .  
3) Deduce that the points  $E, C$  &  $F$  are collinear.
- c. Consider the reference frame defined by  $(A, \vec{i}, \vec{j})$ , where  $\vec{i} = \overrightarrow{AB}$  &  $\vec{j} = \overrightarrow{AD}$ .
  - i. Find coordinates of all points in the given plane.
  - ii. Deduce using coordinates that the points  $E, C$  &  $F$  are collinear.

**IV-** Let  $ABC$  be any triangle and  $x \in \mathbb{Z}^*$  so that  $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AB} + x \cdot \overrightarrow{AC}$  and  $\overrightarrow{AM} = x \cdot \overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC}$ .

- a. Find  $\overrightarrow{EF}$  in terms of  $\overrightarrow{BC}$  &  $x$ .
- b. For what values of  $x$ , the vectors  $\overrightarrow{EF}$  and  $\overrightarrow{BC}$  have the same sense?
- c. Calculate the numerical value of  $x$ , such that  $BCFE$  is a parallelogram.

**V-** Consider the point  $G$ , the centroid of triangle  $ABC$ , and the point  $M$  to be any point in the plane of the given triangle.

- a. Show that:  $\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} = 3\overrightarrow{MG}$ .
- b. Consider the vector:  $\overrightarrow{U} = 3\overrightarrow{MA} + 2\overrightarrow{MB} - 5\overrightarrow{MC}$ .
  - i. Express vector  $\overrightarrow{U}$  independent of  $M$ .
  - ii. Find the set of points  $M$  for  $\|\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC}\| = 6 \text{ units}$ .
- c. Let  $D$  be a point defined by  $\overrightarrow{AD} = -2\overrightarrow{AB} + 3\overrightarrow{AC}$ .
  - i. Show that  $\overrightarrow{BD} = k \cdot \overrightarrow{BC}$ , where  $k$  is a real number to be determined.
  - ii. Deduce that the points  $B, D$  &  $C$  are collinear.

**VI-** Let  $ABC$  be any triangle and  $D$  be a point defined by:  $\overrightarrow{AD} = 3\overrightarrow{AB} - 2\overrightarrow{AC}$ .

- a. Prove that the points  $B, C$  &  $D$  are collinear, then place  $D$ .
- b. Take  $E$  &  $F$  to be any two points defined by:  $\overrightarrow{AE} = \overrightarrow{AC} + 2\overrightarrow{AB}$  and  $\overrightarrow{AF} = 5\overrightarrow{AB} - 8\overrightarrow{AC}$ .  
Express the vectors  $\overrightarrow{DE}$  then  $\overrightarrow{DF}$  in terms of  $\overrightarrow{AB}$  &  $\overrightarrow{AC}$ .
- c. Verify that  $\overrightarrow{DF} + 2\overrightarrow{DE} = \vec{0}$ .

**VII-** Consider the plane of reference  $(O, \vec{i}, \vec{j})$  the vectors  $\vec{V} = -2\vec{i} + 3\vec{j}$  and the points  $A(1; -2), B(3; -4), C(-1; 2)$  and the point  $M$  such that  $\vec{OM} = x\vec{i} + y\vec{j}$ .

- 1) Calculate the coordinates of vector  $\vec{U} = 2\vec{OB} - 3\vec{OC} + 3\vec{BA}$ .
- 2) Find a relation between  $x$  &  $y$  so that the point  $A, B$  &  $M$  are collinear.
- 3) Determine the real values of  $x$  &  $y$  where,  $\vec{AM} = -2\vec{V}$ .
- 4) Find the coordinates of the points  $A, B$  &  $M$  in the system  $(C, \vec{i}, \vec{j})$ .

**VIII-** Consider the plane of reference  $(o, \vec{i}, \vec{j})$  the points  $A, B, C, D$  &  $M$  such that:  $\vec{OA} = -\vec{i} - \vec{j}, \vec{OB} = 4\vec{i}, \vec{OC} = 4(\vec{i} + \vec{j}), \vec{OD} = -\vec{i} + 2\vec{j}$  &  $\vec{OM} = x\vec{i} + y\vec{j}$ .

- a. Show that  $ABCD$  is a parallelogram.
- b. Calculate the coordinates of the point  $E$  so that,  $\vec{AE} = 2\vec{AB}$ .
- c. Find the coordinates of the point  $F$  so that  $BCFD$  is a parallelogram.
- d. Show that the points are collinear.
- e. In the new reference  $(A, \vec{i} + \vec{j}, -\vec{i} + 2\vec{j})$  the coordinates of the point  $M$  becomes  $M(X; Y)$ .
  - 1) Find a relation between coordinates of  $M$  in the two reference frames.
  - 2) Deduce the coordinates of the points  $A, B, C$  &  $D$  in the new system.

Mastering problems		
Chapter	Exercises	Pages
CH-7: Vectors	1, 2 & 4	141
	6	142
	9	143
	15 & 16	145
	18 & 19	146