I- The sum of three consecutive terms of an arithmetic progression is equal to 33, and the sum of squares of these terms is 365.

a. Translate the above text into a system of two equations.
b. Determine the numerical value for each term.

II- Consider the sequence (u) defined by: $\begin{cases} u_0 = -1 \\ u_{n+1} = \dfrac{9}{6-u_n} \quad \text{for all } n \in N \end{cases}$

And the sequence $(V_n)_{n\in N}$ defined by: $v_n = \dfrac{1}{u_n - 3}$.

a. Find the term, v_2. Is (u) an arithmetic sequence? Justify.
b. Show that (V_n) is an arithmetic sequence, whose common difference and first term are to be determined.
c. Deduce the sense of variation of (V_n).
d. Find v_n in terms of n then deduce u_n in terms of n.
e. Calculate, $S = v_1 + v_2 + ... + v_5$.

III- Consider the sequence (U) defined by: $\begin{cases} u_0 = 2 \\ u_{n+1} = \dfrac{1}{4}u_n - 3 \quad \text{for all } n \in N \end{cases}$

And the sequence (V) defined by: $v_n = 2u_n + 8 \quad \forall n \in N$.

1. Compute the numerical values of the terms: $u_1, u_2 \,\&\, u_3$.
2. Determine the nature of the sequence, (U).
3. Show that the sequence (V) is a geometric sequence.
4. Find v_n in terms of n then deduce u_n in terms of n.
5. Determine in terms of n the sum: $s_n = v_0 + v_1 + v_2 + ... + v_n$.
6. Deduce the value of the sum: $t_n = u_0 + u_1 + u_2 + ... + u_n$.

IV- Consider the sequence $\{U_n\}$ defined for all natural numbers n by $U_{n+1} = 4U_n + 2$ and $U_0 = 1$

Part-A: 1) Calculate U_1 and U_2, then verify that $\{U_n\}$ is neither arithmetic nor geometric.

2) Let $W_n = -U_n + K$ find K so that the sequence $\{W_n\}$ is geometric.

Part-B: Let $\{V_n\}$ be a sequence defined for all natural numbers n by $V_n = -U_n - \dfrac{2}{3}$

1) Show that $\{V_n\}$ is a geometric sequence whose common ratio and first term are to be found.
2) Express V_n then U_n in terms of n.
3) If $S = V_0 + V_1 + ... + V_n$, calculate S in terms of n and deduce $\sum_{p=0}^{n}(U_p + 2p - 3)$ in terms of n.

V- Calculate the sum: $S = 1 + x + x^2 + x^3 + x^4 + ... + x^{11}$.

VI- Consider the sequence (U_n) defined by: $\begin{cases} U_0 = 2 \\ U_{n+1} = \frac{1}{2}U_n + 3 \end{cases}$

1. Calculate $U_1 \& U_2$, then verify that (U_n) is neither arithmetic nor geometric.
2. Let $V_n = U_n - 6$
   a. Show that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b. Calculate V_n & U_n in terms of n.
3. Calculate the sum $S_n = V_0 + V_1 + ... + V_n$ in terms of n & deduce $S'_n = U_0 + U_1 + ... + U_n$.

VII- Let (U_n) be a sequence defined, for every natural number n, by: $\begin{cases} U_0 = 1 \\ U_{n+1} = U_n + \frac{1}{2^n} \end{cases}$.

1) Calculate, $U_1 \& U_2$.
2) Show that the sequence (U_n) is neither arithmetic nor geometric.
3) Calculate $U_{n+1} - U_n$. Deduce that the sequence (U_n) is strictly increasing.
4) Consider the sequence (V_n) defined by: $V_n = U_{n+1} - U_n$.
   a) Verify that (V_n) is a geometric sequence whose common ratio r and first term V_0 are to be determined.
   b) Express V_n in terms of n, then deduce the value of V_{10}.
   c) Calculate, in terms of n, the sum $S = V_0 + V_1 + V_2 + ... + V_n$.

VIII- In year 2010, the annual cost of the participation of a member in a sport club is 1 000 000 LL. This cost increases 10% annually. Starting from the second year, a participant will get an annual reduction of 50 000 LL. Designate by C_n the annual cost of a member in year (2009 + n). In this case, C_1 = 1000 000 LL

1) Prove that C_2 = 1 050 000 LL.
2) Verify that $C_{n+1} = 1.1C_n - 50\,000$, where $n \geq 1$.
3) Consider the sequence (V_n) defined by: $V_n = C_n - 500\,000$.
   a) Prove that (V_n) is a geometric sequence of common ratio 1.1.
   b) Express V_n in terms of n, then deduce C_n in terms of n.
   c) Find the annual cost of the participation of a member in year 2020.
4) Let $S_n = C_1 + C_2 + \ldots + C_n$ and $R_n = V_1 + V_2 + \ldots + V_n$, where $n \geq 1$.
   a) Write R_n in terms of n.
   b) Deduce that $S_n = 5\,000\,000\,(1.1^n - 1) + 500\,000\,n$.