AlMahdi High Schools — *Mathematics* — *10^{th}-Grade*

Name: . . . . — *"Powers And Radicals"* — *W.S-6.*

Exercise 1: ***The following parts are independent*.**

1. Let $A=\sqrt{2}\left(\sqrt{2+\sqrt{3}}\right)\left(1-\sqrt{3}\right)$. Calculate A^2 then deduce the value of A.

2. Find nature of triangle ABC such as : $AB=\dfrac{\sqrt[5]{\sqrt{2^{11}}}}{\sqrt[5]{8}}$, $AC=\sqrt{18}-\sqrt{8}$ and $BC=\dfrac{\sqrt{2^{21}}\times\sqrt[3]{125}}{5\left(2^3+2^3+2^3+2^3\right)^2}$.

3. Simplify the expression: $\sqrt[4]{(a-b)^4}+\sqrt{(a-b)^2}+2\sqrt[6]{(a-b)^6}$ where $a<b<0$

Exercise 2: Given $A=\left(\sqrt[6]{\sqrt{5}-1}\right)\left(\sqrt[6]{\sqrt{5}+1}\right)\left(\sqrt[3]{4}\right)$ & $M=\dfrac{27^n-9^{2n}}{3^n-9^n}$, where n is a real number

1) Show that A is equal to a natural number to be determined.
2) Show that $M=3^{\alpha n}$ where α is an integer to be determined.
3) Find the value of n for which $M=\left(\dfrac{1}{9}\right)^{2n-1}$.

Exercise 3: Given $A=\dfrac{2^n+2^{-n}}{2}$ and $B=\dfrac{2^n-2^{-n}}{2}$

1. Prove that $A^2-B^2=1$
2. Show that: $\sqrt{2+\sqrt{2}}.\sqrt{2+\sqrt{2+\sqrt{2}}}.\sqrt{2-\sqrt{2+\sqrt{2}}}=\sqrt{2}$

Exercise 4:

1- Calculate the product of: $\left(1+\sqrt{5}\right)\left(1+\sqrt{2}\right)$
2- Rationalize the denominator of: $\dfrac{4}{1+\sqrt{2}+\sqrt{5}+\sqrt{10}}$
3- Verify that: $\sqrt{\dfrac{x+\sqrt{x^2-2}}{2}}+\dfrac{\sqrt{x-\sqrt{x^2-2}}}{2}=\sqrt{x+\sqrt{2}}$ $(x>2)$.

Exercise 5: Let $A=\dfrac{9^n+6.3^n+9}{9^n+3^{n+1}}$

a) Show that: $A=3^{1-n}+1$.
b) Find the value of n so that $A=28$.

Exercise 7: For each question in the table below, choose with justification the only correct answer.

No.	*Questions*	*Expected Answers*		
		a	*b*	*c*
1.	$\sqrt{-x-1}$ exists for	$x \leq 1$	$x > 0$	$x \leq -1$
2.	The solution set, S, of $\frac{\lvert 3x-6\rvert}{(x-1)^2} \geq 0$, is	ϕ	$]-\infty;1[\cup]1;+\infty[$	$]1;+\infty[$
3.	If $E=\left(5+\sqrt{5}\right)\left(\sqrt{5}-\sqrt[4]{5}\right)\left(\sqrt{5}+\sqrt[4]{5}\right)$ then	$E = 0$	$E = 20$	$E = 30 + 10\sqrt{5}$
4.	$\sqrt[3]{\left((-5)^2\right)^3} =$	10	-25	25
5.	If $x \in \mathbb{R}^-$ then, $\sqrt{x^4+4x^2} =$	$x^2 + 2x$	$x\sqrt{x^2+4}$	$-x\sqrt{x^2+4}$
6.	The solution set S of the equation: $\lvert x+3\rvert+1=2x+1$ is	$S=\{3,-1\}$	$S=\{3\}$	$S=\{\frac{1}{3},-1\}$
7.	If $x \in \left[-1;\ 5\right]$ then	$\lvert x-2\rvert \leq 3$	$\lvert x+2\rvert \leq 3$	$\lvert x+2\rvert < 3$
8.	If $a>0$ & $b<0$, then $\frac{\sqrt{a^2b^2}+2\lvert ab\rvert}{\sqrt[3]{a^3b^3}} =$	-3	3	0
9.	If $E=\sqrt[6]{(3-\pi)^6}-\sqrt[5]{(\pi-4)^5}-\sqrt{(4-\pi)^2}$ Then E =	$\pi-3$	$-(\pi+3)$	$3-\pi$
10.	If $x=\sqrt{2}\times\sqrt{2-\sqrt{3}}(1+\sqrt{3})$, then $x=$	-2	2	$\sqrt{3}-1$

Exercise 9: ***The parts of this question are independent.***

1. Let $A=\frac{2^{4n}+6.2^{3n}+9.2^{2n}}{2^{3n}+3.2^{2n}}$; $(n \in Z)$
   a) Show that : $A=2^n+3$.
   b) Find n so that: $A=\frac{97}{32}$.
   c) Determine the value of m so that: $\frac{4^{m^2}}{8^m}=2^m$
2. Given $a=\sqrt{3+2\sqrt{2}}\left(2\sqrt{2}-3\right)\left(\sqrt{10}+\sqrt{5}\right)$.
   Calculate a^2, then deduce the value of a.

Exercise 10: Express without radical sign:

a) $\sqrt[6]{(x-1)^6}$ c) $\sqrt[4]{625a^8}$ e) $\sqrt[5]{-243b^5}$ g) $\left(\sqrt{a^3}\right)^2$ s.t $a>0$.

b) $\left(\sqrt[4]{9x^2}\right)^2$ d) $\left(\sqrt[8]{x^2}\right)^4$ f) $\left(\sqrt[3]{2a^2}\right)^3$ h) $\left(\sqrt[4]{y^2}\right)^6$.

Exercise 11: Simplify:

1) $\sqrt[10]{x^4}.\sqrt[10]{x^3}.\sqrt[10]{x^3}$ 3) $\sqrt[4]{\frac{81}{16}}$ 5) $4\sqrt[4]{5a^2}.8\sqrt[4]{125a}.2\sqrt[4]{a}$. Where $a\geq 0$

2) $\frac{\sqrt[4]{32}}{\sqrt[4]{2}}$ 4) $\frac{\sqrt[3]{16a^5b^3}}{\sqrt[3]{2a^2}}$ 6) $\frac{12\sqrt[8]{3x^{10}}}{\sqrt[8]{3x^2}}$.

Exercise 12: Solve the following equations then check:

a- $\left(4^{3x+1}-\frac{1}{2}\right)\left(25^{x-1}-\sqrt[5]{5}\right)=0$ c- $\sqrt[3]{2x-1}=-3$ e- $\sqrt{x+2}-\sqrt{x-1}=1$

b- $\sqrt[4]{32-x^4}=x$ d- $\sqrt{5-\sqrt{3x-2}}=1$ f- $\sqrt[4]{5x-5}+11=6$

Exercise 13: Simplify or (Express in form of a single radical)

$\sqrt[4]{2}.\sqrt{3}$; $\sqrt[3]{a^2}.\sqrt[4]{a^3}.\sqrt[5]{a}$; $\frac{\sqrt[3]{5}}{\sqrt[4]{5}}$.

$\sqrt[7]{b^3}.\sqrt[3]{b^2}.\sqrt{b}$; $\frac{\sqrt[4]{27}}{\sqrt{3}}$; $\frac{\sqrt[5]{x^4}}{\sqrt[3]{x}}$.

Exercise 14: Rationalize the denominator of:

1- $\frac{\sqrt[3]{54x^3y}}{\sqrt[3]{3x}}$ 3- $\sqrt[3]{\frac{5}{2a^2}}$ 5- $\frac{2}{\sqrt[3]{2}-\sqrt[3]{5}}$

2- $\frac{2}{\sqrt{x^5}}$ 4- $\frac{2}{1+\sqrt[3]{a}}$ 6- $\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}$.

Exercise 15: Compute the following products:

No.	Product form	Expanded form	The conjugate of
1)	$\left(a\sqrt{b}+c\sqrt{d}\right)\left(a\sqrt{b}-c\sqrt{d}\right)$		$\left(2\sqrt{3}-5\sqrt{2}\right)$ *is*
2)	$\left(\sqrt[n]{x^k}\right)\left(\sqrt[n]{x^{n-k}}\right)$		$\sqrt[5]{x^2}$ *is*
3)	$\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)$		$\sqrt[3]{5}-\sqrt[3]{7}$ *is*
4)	$\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)$		
5)	$\left(\sqrt[4]{a}-\sqrt[4]{b}\right)\left(\sqrt[4]{a^3}+\sqrt[4]{a^2b}+\sqrt[4]{ab^2}+\sqrt[4]{b^3}\right)$		
6)	$\left(\sqrt[4]{a}+\sqrt[4]{b}\right)\left(\sqrt[4]{a^3}-\sqrt[4]{a^2b}+\sqrt[4]{ab^2}-\sqrt[4]{b^3}\right)$		

Recall that

I- To reduce several radicals, with ***positive*** radicands, to the same index:

a) Find the LCM of the given indices.
b) Go as you do while taking a common denominator.

II- Two radicals are ***identical*** if, in simplest form, they have the ***same index*** and ***same radicand***.

III- ***Addition*** and ***subtraction*** of radicals can be performed only if radicals are ***identical***.

IV- Two radicals are conjugate of each other if their product contains no radicals when expressed in simplest form.