Lycée Des Arts | Mathematics | 9^{th}-Grade

Name: . . . . | "Thales' Property" | W.S-6

1) Find the numerical value of the strictly positive real number x in each of the following cases:

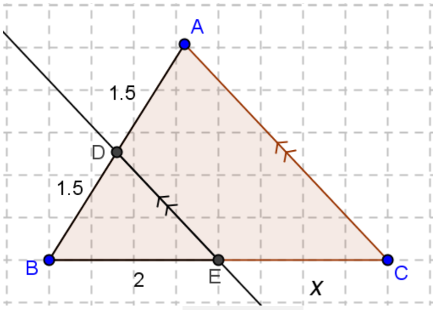


Fig-1.

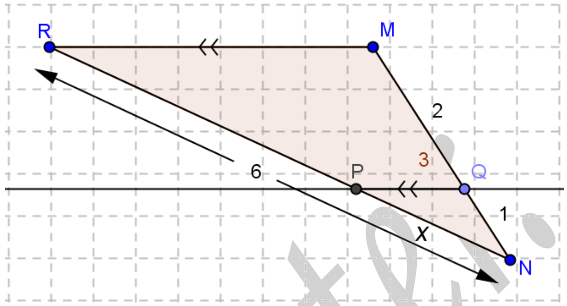
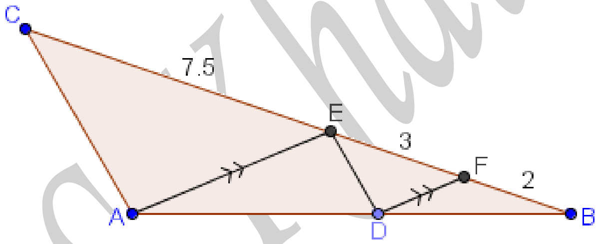


Fig-2.

2) In triangle ABC, we have $(AE)//(DF)$.

Prove that $(AC)//(DE)$.

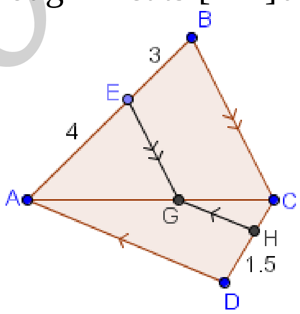


3) Let RNK be any triangle. The parallel drawn to (NK) intersects sides $[RN]\&[RK]$ at E and F respectively. The parallel to (NF) drawn through E cuts $[RK]$ at G.

Prove that: $RF^2 = RG \times RK$.

4) Consider the two triangles $ABC \& ACD$, where $(EG)//(BC)\&(GH)//(AD)$.

Compute the length of $[CH]$.



5) Given a semi-circle (c) of center O and diameter $BC = 6cm$, and A is a point on (c), such that $AB = 3cm$. Let I be a point on $[BC]$ such that $BI = \frac{2}{3}BC$, and J is a point on $[AB]$ such that $BJ = 2cm$. Draw a sketch

a. Prove that (IJ) is parallel to (AC). Deduce relative position of (IJ) and (AB).

b. If N is the midpoint of $[BI]$. Prove that $\frac{JN}{BI} = \frac{AO}{BC} = \frac{1}{2}$, and that $(JN)\|(AO)$.

6) Let $RIDA$ be a parallelogram and M be a point on $[IA]$. $[RM)$ cuts $[ID]$ at O and $[DA]$ at N.

Prove that: $RM^2 = MN \times MO$.

7) Consider the right angle $u\hat{O}v$. Let $R \& N$ be two points on $[Ou)\&[Ov)$ respectively such that $OR = 3cm$ and $ON = 4cm$.

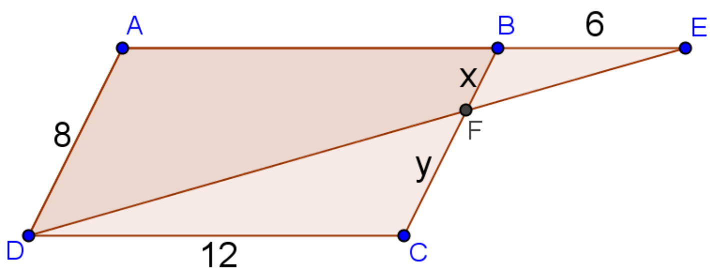
a. Compute RN.

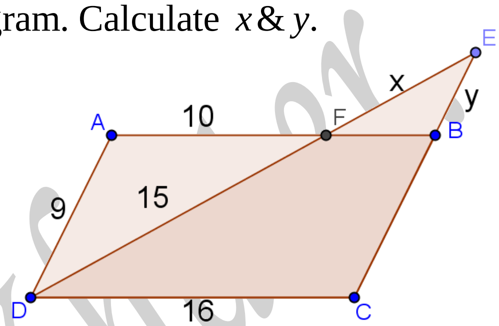
b. Let K be a point on $[RN]$ such that $RK = 3cm$. The circle of diameter $[NK]$ intersects $[ON]$ at F. Calculate NF and FK.

8) Let $M, N \& Q$ be three collinear points taken in this order, such that $MN = 6cm$ and $NQ = 4cm$. (C) is the circle of diameter $[MN]$ and (C') the circle of diameter $[NQ]$. A is a point on (C), so that $MA = 2cm$. (AN) intersects (C') at B.

a. Calculate measure of AN.
b. Show that (MA) *and* (QB) are parallel.
c. Calculate the lengths of $QB \& BN$.

9) In each of the following figures, $ABCD$ is a parallelogram. Calculate $x \& y$.



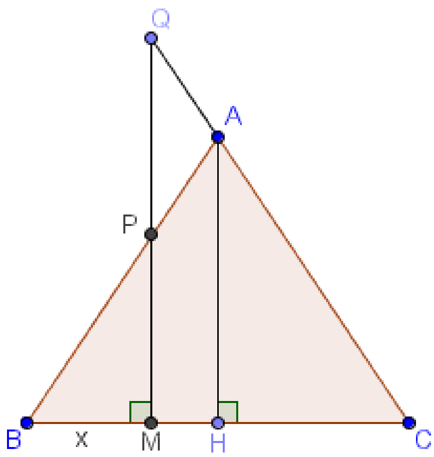


10) Consider a circle (C) of center O and diameter $AB = 12cm$. E is a point of $[OB]$ such that $OE = 4cm$. Draw the circle $C'(E; BE)$. (C') intersects $[OB]$ again at point N.

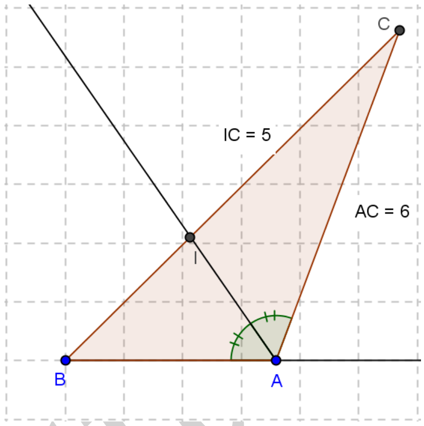
a. Draw a figure.
b. Let M be a point of (C') such that $BM = 2cm$ and the line (BM) cuts (C) at P, where (PO) cuts (C) at K.
   i. What is the nature of triangle MNB? Deduce the length of MN.
   ii. Prove that (MN) *and* (AP) are parallel.
   iii. Deduce the length of BP.
c. Prove that $(ME)//(PO)$.
d. *i*) Calculate the ratio $\frac{BN}{BO}$, and deduce the position of the point N in the triangle PBK.
   ii) (PN) cuts (BK) in I. Prove that I is the midpoint of $[BK]$.
   iii) (KN) cuts (PB) in J. Prove that $IJ = 6cm$.

11) ABC is an isosceles triangle at vertex A. The altitude issued from point A cuts $[BC]$ at H. (See figure below)
Given $BC = 6cm$ and $AH = 4cm$.
Let M be a point of [BH] such that $BM = x$. The parallel to (AH) through M cuts (AB) at P and (AC) at Q.

1) a- Calculate BH and give an encirclement for x.
   b- Show that: $\frac{MP}{AH} = \frac{x}{3}$. Deduce MP in terms of x.
2) a- Express MC in terms of x.
   b- Show that $MQ = \frac{4}{3}(6 - x)$.
   c- Find the value of x so that $MQ = 3MP$.
   d- In this case, precise the position of P on $[AB]$.



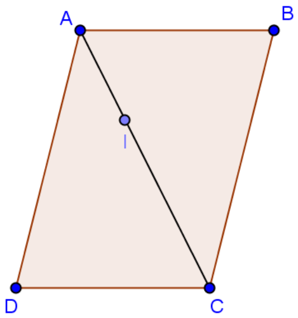
12) In the adjacent figure $[AI)$ is the bisector of the angle $B\hat{A}C$, where $IC = 5cm$, $AC = 6cm$ & $BC = 8cm$.



a) Reproduce the figure.
b) Explain how can you use the segment $[BC]$ and the point I to **locate** the point D on $[BA)$ so that, $BD = \frac{8}{3}BA$.
c) What is the relative position of (AI) & (DC)? Justify.
d) From this part on we admit that $AI = b, DC = a$ and that the straight lines (AI) & (DC) are parallel.
   i. Code the figure.
   ii. Show that the triangle ACD is isosceles.
   iii. Prove that: $\frac{AB}{AC} = \frac{IB}{IC}$.
   iv. Show that: $b = \frac{3}{8}a$
   v. If the perimeter of the trapezoid $AICD = 19.25cm$, then deduce that $a = 6cm$ and $b = 2.25cm$.

13) The unit of length in this exercise is the centimeter, where the ***parts A and B are independent***.

Let I be any point on the diagonal $[AC]$ of the parallelogram $ABCD$ of sides $AB = x + 5$ and $AD = 4x + 2$. $[DI)$ cuts $[AB]$ and (BC) in points N & O respectively. Complete the figure.



Part-A:

1) If $BN = 2cm$, then:
   a. Prove that $\frac{ID}{IN} = \frac{x+5}{x+3}$.
   b. Calculate the numerical value of x, if $\frac{IC}{IA} = \frac{3}{2}$
2) Prove that $DI^2 = IN \times IO$.

Part-B:

The parallels drawn from I to the straight lines, (BC) and (DC), cut $[AB]$&$[AD]$ in the points E & F respectively. Complete the figure.

1) If $FI = 2cm$ & $IE = x\,cm$, then find the ratio of AI to AC in terms of x.
2) Prove that: $\frac{FI}{DC} = \frac{IE}{CB}$
3) Show that: $x^2 - 3x - 4 = (x+1)(x-4)$
4) Use parts 2 & 3 to calculate the value of x.
5) Prove that (FE) and (DC) are parallel.