Al- Mahdi High	*Mathematics*	*11th-Grade*
Name:	*Rational functions*	*W.S-5*

I- Consider the function *f* defined on an interval $I = \mathbb{R} - \{2\}$ by $f(x) = \frac{x^2 - 3x + 6}{x - 2}$ and its representative curve (C).

1- Let the straight line (Δ) be the oblique asymptote of (C).
   a. Prove that the straight line (Δ) is of equation: $y = x - 1$.
   b. Study the relative positions of (C) with respect to (Δ).

2- Let the straight line (l) be the vertical asymptote of (C).
   a. Find the equation of (l).
   b. Prove that $S(x_S; y_S) = (\Delta) \cap (l)$ is the center of symmetry of (C).

3- Verify that $f'(x) = \frac{x(x-4)}{(x-2)^2}$ and set up the table of variations of *f*.

4- Plot S, then construct $(l), (\Delta) \& (C)$.

5- Discuss according to the values of m the number of solutions of: $f(x) = m(x-2) + 2$

6- Let g be a function so that $g(x) = \frac{x^2 - 3x + 6}{|x - 2|}$ of representative curve (C_1).
Use (C) to construct (C_1) and set up the table of variations of g.

II- Consider the function *f* defined over $\mathbb{R}$ by: $(x) = \frac{-x^3 + 5x}{x^2 + 3}$, and let (C) be its representative curve in an orthonormal system $(O\,; \vec{\imath}; \vec{\jmath}\,)$.

1) a. Verify that: $(x) = -x + \frac{8x}{x^2+3}$.
   b. Show that *f* is an odd function, then deduce the element of symmetry of (C).

2) a. Show that: $f'(x) = \frac{(x^2+15)(1-x^2)}{(x^2+3)^2}$.
   b. Draw the table of variations of *f*.

3) Let (d) be the straight line of equation: $y = -x$.
   Show that (d) is an asymptote to (C), then study the relative positions of (C) and (d).

4) Write an equation of the tangent (T) to (C) at the point of abscissa $x = 0$.

5) a. Calculate $f(-3)$ and determine the coordinates of the points of intersection of (C) with the abscissas axis.
   b. Trace $(d), (T) \& (C)$.

6) Solve graphically: $f(x) < 1$.

III- Choose with ***justification*** the only correct answer:

No.	Propositions	Expected answers		
		a	b	c
1.	$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}=$	$+\infty$	$-\infty$	$\frac{1}{4}$
2.	If $f(x)=x^3+3x^2+px$, then f is strictly increasing on $\mathbb{R}$ for	$p\geq 3$	$p<3$	$-1<p<0$
3.	If $f(-x)+f(x)=2$ for every $x\ \&-x$ belong to D_f then the curve (C_f) admits a center of symmetry:	O(0,0)	I(0,1)	J(1,0)
4.	The function f defined over [-2,1] by $f(x)\frac{\lvert x\rvert}{x^2+4}$ is:	Odd	Even	Neither even nor odd

IV- Let f be a function defined over $\mathbb{R}$ by: $f(x)=\frac{x^3-2x^2+5x-2}{x^2+1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Express $f(x)$ in form of $ax+b+\frac{cx}{x^2+1}$, where $a,b\ \&\ c$ are non-zero integers.
2) Form this part on take $a=1, b=-2\ \&\ c=4$
   a. Express $f(x)$ in the new form, then determine: $\lim_{x\to+\infty}f(x)$ and $\lim_{x\to-\infty}f(x)$.
   b. Show that the straight- line (l) of equation $y=x-2$ is an asymptote to (C).
   c. Study the relative position of (C) and (l).
3) Given that (C) cuts the $x-axis$ at a point B of abscissa $\alpha\in]0;0.5[$
   a. Prove that: $f'(x)=\frac{x^4-2x^2+5}{(x^2+1)^2}$, and then set up the table of variations of f.
   b. Verify that B is unique, then study the sign of $f(x)$ in terms of α.
   c. Prove that $I(0;-2)$ is a center of symmetry for (C).
4) Determine the coordinates of the points $A\ \&\ B$ of (C), where the tangent is parallel to the oblique asymptote (l) and $x_A<x_B$.
5) Plot the points $I, A\ \&\ B$, and trace the curves (l), the tangents at $A\ \&\ B$ and (C).
6) Solve graphically: $x-1<f(x)<x$.
7) Deduce from (C) the curve (C') representing the function h defined by $h(x)=f(|x|)$.