Lycée Des Arts
Name: . . . . . . . . .

Mathematics
$9^{\text {th }}$-Grade
"Vector Translations"

1- Answer by True or False, and correct false statements:
a. A straight line admits one director vector only.
b. There is only one vector parallel to $\vec{V}(2,1)$.
c. If $\overrightarrow{A B}+\overrightarrow{B C}=\overrightarrow{A C}$ then it is necessary that the points $A, B \& C$ are collinear.
d. If $A B C D$ is a parallelogram, then $\overrightarrow{A B}=\overrightarrow{C D}$
$\boldsymbol{e}$. If $I$ is the midpoint of $[A B]$ and $M$ is any point in the plane, then $\overrightarrow{M A}+\overrightarrow{M B}=2 \overrightarrow{M I}$
2- For each statement, indicate with justification the only correct answer. (3-pts)

| $\mathcal{N}$ o. | Statements |  | Proposed answers |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  | $a$ | 6 | $c$ |  |
| 1. | The vector $\vec{u}(3,4)$ is parallel to the <br> straight line $(d)$ of equation, | $3 x+4 y-1=0$ | $4 x+3 y-1=0$ | $4 x-3 y-1=0$ |  |
| 2. | The vector $\vec{u}(2,3)$ is collinear with | $\vec{v}(-4,-6)$ | $\vec{v}(-4,6)$ | $\vec{v}(4,-6)$ |  |
| 3. | A zero vector is a vector with zero <br> magnitude and | No direction <br> No sense | Infinite directions <br> No sense | No direction <br> Infinite senses |  |
| 4. | If $\vec{v}=\vec{u}+\vec{v}$, then $\\|\vec{w}\\|=$ | $\\|\vec{u}\\|+\\|\vec{v}\\|$ | $\\|\vec{u}+\vec{v}\\|$ | $\sqrt{\vec{u}+\vec{v}}$ |  |

3- Given that triangle $Q M N$ is isosceles of principal vertex $Q$.
a. Construct the point $I$ such that: $\overrightarrow{Q I}=\overrightarrow{Q M}+\overrightarrow{Q N}$.
b. Prove that point $I$ is the image of the point $N$, by a translation vector to be determined.
c. Determine the nature of quadrilateral QMIN. Justify.
$d$. Let $J$ and $K$ be the respective symmetries of $I$ with respect to $M$ and $N$.
Show that $Q$ is the midpoint of $[J K]$.(Use vectors)
4- Use the adjacent figure to complete and justify the following equalities:

1. $\overrightarrow{A B}+\overrightarrow{B I}+\overrightarrow{I C}=\ldots$.
2. $\overrightarrow{A B}+\overrightarrow{A C}=\ldots$.
$3 . \overrightarrow{I B}+\ldots . \overrightarrow{0}$
3. $\overrightarrow{A B}+\overrightarrow{I C}=\overrightarrow{A \ldots}$.


5-Consider the points $A(-1 ; 3), B(2 ; 1), C(1 ;-1)$, and the vector $\vec{v}(a-1 ; b+2)$. Calculate the numerical value of:
a. $a \& b$, if $\vec{v}=2 \overrightarrow{A B}-\overrightarrow{B C}$.
b. $b$ so that $\vec{v}$ is parallel to the ordinate axis.

6- In the plane of orthonormal system of axes $x^{\prime}$ Oxand $y^{\prime} O y$, plot the points: $A(-2 ; 4), B(-5 ;-2) \& C(0 ; 3)$.
a. Determine both graphically and analytically the coordinates of $\overrightarrow{A B}$.
b. Let $E$ be the translate of $C$ by the translation of $\overrightarrow{A B}$.
$i$. Plot then find the coordinates of the point $E$.
ii. Calculate coordinates of vectors: $\overrightarrow{A E} \& \overrightarrow{A C}$, then deduce that: $\overrightarrow{A E}=\overrightarrow{A B}+\overrightarrow{A C}$.
c. What is the nature of quadrilateral $A B E C$ ? Justify your answer.

7- In the plane of orthonormal system of axes x'Oxand y'Oy, place the points: $A(1 ;-1), B(3 ; 1) \& C(-1 ; 3)$.
a. Determine the nature of triangle $A B C$.
b. Construct the point $D$ so that: $\overrightarrow{B D}=\overrightarrow{B A}+\overrightarrow{B C}$, then determine its coordinates.
c. Show that $(B D)$ is parallel to $x^{\prime} O x$, then compute the area of triangle $B C D$.

8 - Consider the point $M$, the midpoint of $[B C]$, in triangle $A B C$.
a. If $G$ is the centroid of triangle $A B C$ then,
i. Prove that: $\overrightarrow{G B}+\overrightarrow{G C}=2 \overrightarrow{G M}$.
ii. Deduce that: $\overrightarrow{G A}+\overrightarrow{G B}+\overrightarrow{G C}=\overrightarrow{0}$.
b. Let $I$ be any point in the given plane. Prove that $\overrightarrow{I A}+\overrightarrow{I B}+\overrightarrow{I C}=3 \overrightarrow{I G}$.

9- Let $M$ and $N$ be the respective midpoints of $[B C] \&[A M]$ in triangle $A B C$. ( $B N$ ) cuts $[A C]$ at a point $I$, the parallel drawn through $A$ to $(B N)$ cuts $(C B)$ at a point $E$.
a. Draw a figure.
b. Prove that $B$ is the midpoint of $[E M]$.
c. Use Thales' property to prove that: $\frac{A I}{A C}=\frac{1}{3}$.
d. Show that: $\overrightarrow{A E}+\overrightarrow{M C}=\overrightarrow{A B}$.

10- Given a triangle $A B C$.
a. Plot the points $M$ and $N$ so that: $\overrightarrow{A M}=\overrightarrow{B C}$ and $\overrightarrow{N B}=\overrightarrow{A C}$.
b. Show that $\overrightarrow{N A}=\overrightarrow{B C}$.
c. Deduce that $A$ is the midpoint of $[M N]$.

11- Given a circle $C(O ; R c m)$ and diameter $[B C]$, let $A$ be a point of ( $C$ ) so that $C A=R \mathrm{~cm}$.

1) $\boldsymbol{a}$ - Draw a figure then locate $D$, the image of $B$ by the translation of $\overrightarrow{A C}$.
$\boldsymbol{b}$ - Determine the nature of quadrilateral $A B D C$ ? Justify your answer.
2) $i$ - Locate the point $M$ so that; $\overrightarrow{C M}=\overrightarrow{C O}+\overrightarrow{C A}$.
ii- Locate the point $N$ so that; $\overrightarrow{O N}=\overrightarrow{O A}+\overrightarrow{O C}$.
iii- Deduce that $\overrightarrow{D N}+\overrightarrow{C A}=3 \overrightarrow{O A}$.
3) Prove that $A$ is the midpoint of $[M N]$.

12- Let $I$ be a point on a circle (c) with diameter $[E F]$. And designate by $J$ the symmetric of $E$ with respect to $I$.
a. Prove that triangle EFJ is isosceles.
b. Let $K$ be the image of $I$ by the translation of $\overrightarrow{E F}$. Compare the vectors $\overrightarrow{I J} \& \overrightarrow{F K}$.
c. Deduce the nature of quadrilateral $I F K J$.
d. Construct $G$ such that $\overrightarrow{E G}=\overrightarrow{F E}+\overrightarrow{F K}$.
$\boldsymbol{e}$. What does point $I$ represent with respect to $[G K]$ ? Justify your answer.
13- Locate the missing points on the adjacent grid so that:
a) $\overrightarrow{A R}=\overrightarrow{B A}+\overrightarrow{B C}$.
b) $\overrightarrow{B N}=-\overrightarrow{C A}$.
c) $\overrightarrow{A J}=\overrightarrow{A N}+\overrightarrow{A C}$.
d) $\overrightarrow{A I}+\overrightarrow{B I}=\overrightarrow{0}$.

14- $A B C D E F$ is a regular hexagon with center $O$.
i. Correct false statements if there is any:

|  | True | False | Correction |
| :---: | :---: | :---: | :---: |
| $\overrightarrow{A F}=\overrightarrow{B O}$ |  |  |  |
| $\overrightarrow{O A}=\overrightarrow{O B}$ |  |  |  |
| $\overrightarrow{A F}+\overrightarrow{F O}=\overrightarrow{B C}$ |  |  |  |
| $\overrightarrow{A B}+\overrightarrow{B C}=\overrightarrow{F D}$ |  |  |  |

ii. Complete the following:
a. $\overrightarrow{F E}=\overrightarrow{A_{1} \ldots}=\overrightarrow{O \ldots}=\overrightarrow{B_{\ldots} \ldots}$.
$\overrightarrow{O E}=\vec{O} \ldots+\vec{D}$.
$\overrightarrow{O F}+\overrightarrow{O D}=\overrightarrow{. . O}$.

b. The translate of $B$ by the translation with vector $(\overrightarrow{A F}+\overrightarrow{F E})$ is the point $\ldots \ldots$
c. F is the translate of O by the translation with vector... $\vec{A}$.

15- Consider the parallelogram $A B C D$.
a. Construct the points $E, F, G$ and $H$ defined by:

$$
\overrightarrow{D E}=\frac{5}{4} \overrightarrow{D A} ; \overrightarrow{A F}=\frac{6}{5} \overrightarrow{A B} ; \overrightarrow{B G}=\frac{5}{4} \overrightarrow{B C} ; \overrightarrow{C H}=\frac{6}{5} \overrightarrow{C D} .
$$

b. Express the following vectors as a linear combination of $\overrightarrow{A B} \& \overrightarrow{A D}$.

$$
\text { i. } \overrightarrow{E F} . \quad \text { ii. } \overrightarrow{H G} .
$$

c. Deduce the nature of quadrilateral $E F G H$.

16- Consider the parallelogram $A B C D$, so that $A B=6 \mathrm{~cm} \& B C=2 \mathrm{~cm}$. $I$ and $J$ are two points of $[A B]$ so that $A I=I J=2 \mathrm{~cm}$.
$K$ and $L$ are two points of [CD] so that $D L=L K=2 \mathrm{~cm}$.

Indicate the correct answer(s).


|  | $a$ | $b$ | $c$ | $d$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overrightarrow{A I}=\ldots$. | $\overrightarrow{K C}$ | $\overrightarrow{A D}$ | $\overrightarrow{D L}$ | $\overrightarrow{J B}$ |
| $\overrightarrow{A L}=\ldots$ | $\overrightarrow{A I}+\overrightarrow{J C}$ | $\overrightarrow{D A}+\overrightarrow{D L}$ | $\overrightarrow{I J}+\overrightarrow{J K}$ | $\overrightarrow{A D}+\overrightarrow{D L}$ |
| $\overrightarrow{A J}=\ldots$ | $\overrightarrow{K B}+\overrightarrow{A L}$ | $\overrightarrow{I J}+\overrightarrow{A L}$ | $\overrightarrow{A I}+\overrightarrow{D L}$ | $2 \overrightarrow{L C}$ |
| $\overrightarrow{I L}=\ldots$ | $\overrightarrow{D A}$ | $\overrightarrow{B C}$ | $\overrightarrow{J C}$ | $\overrightarrow{I K}$ |

17- In an orthonomral system of axes $x$ 'ox, $y$ 'oy, consider the points $A(5 ; 3), C(-3 ;-3)$ and the circle ( $n$ ) of diameter $[A C]$ and center $I$.

1) Plot the points $\boldsymbol{A}$ and $\boldsymbol{C}$, and draw the circle ( $n$ ).
2) Determine the coordinates of the point $\boldsymbol{I}$ and the radius of the circle (n). (1pt)
3) Consider the point $\boldsymbol{B}(-3 ; 3)$
$\boldsymbol{i}$. Prove that the straight lines $(\boldsymbol{A B}) \&(\boldsymbol{C B})$ are parallel to the coordinate axes, then find their equations.
ii. Deduce the nature of triangle $A B C$.
iii. Show that $\boldsymbol{B}$ belongs to the circle ( $n$ ).
4) Determine the equation of the tangent $(\boldsymbol{d})$ to the circle $(n)$ at $\boldsymbol{B}$.
5) Let $\boldsymbol{S}$ be the translate of $\boldsymbol{I}$ by the vector translation $\overrightarrow{B I}$.
i. Construct $\boldsymbol{S}$ then calculate its coordinates.
ii. Deduce the nature of the quadrilateral $\boldsymbol{A B C S}$.
6) Find the equation of ( $\mathrm{d}^{\prime}$ ) the translate of (d) under the translation with vector $\overrightarrow{\mathrm{BC}}$.

18- In the reference frame $x^{\prime} O x, y^{\prime} O y$ consider the points $R(2,3), S(c-2,3) \& N(-1,2)$, the vector $\vec{u}(2-a, b+1)$ the family of the straight lines: $\left(d_{m}\right): x-2 m x+y=m+1$

1. Determine the numerical values of $a \& b$, so that $\vec{u}=2 \overrightarrow{R N}-3 \overrightarrow{N S}$ where, $c=2$.
2. Determine the locus of the point $S$ as $c$ varies.
3. Determine the real value of $m$, if $\left(d_{m}\right)$ :
a. Passes through the point $A(-2,1)$.
b. Is parallel to straight line $(l):-3 x+y+1=0$
4. Find the equation of the straight line $(\Delta)$ the image of $(R N)$ by the translation of $\overrightarrow{N A}$.
